Analysis of Systematic Risks in Multi-Name Credit and Equity Markets

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Abstract

In this paper, we present an intensity-based common factor model that is used to analyze the valuation of common systematic risks in multi-name credit and equity markets. In particular, we use a hybrid intensity model to price single-name credit instruments such as credit default swaps (CDSs), multi-name credit derivatives such as collateralized debt obligations (CDOs), and equity index options such as calls and puts on the S&P 500.

Once we have the expressions for the model prices of these instruments, we then calibrate the model parameters to fit the market data. We study two problems, the "forward" and "backward" problems: in the former, we start from equity index options and then compute the CDO tranche spreads, while in the latter, we fit the parameters to the CDO tranche spreads and then compute the equity index option prices and implied volatilities. We find that, based on our hybrid model, the systematic risks in the two markets were similar from 2004 to 2007, while the credit market incorporated far greater systematic risk than the equity market during the financial crisis from 2008 to 2010.

1 Introduction

In this paper, we examine the link between equity and credit markets by considering an intensity-based model that can price both equity index options and CDOs. We employ a *hybrid equity-credit* intensity model for examining and comparing the systematic risks in the two markets before and during the financial crisis.

Recently, there have emerged *structural-based* hybrid models for pricing equity derivatives and *multi-name* credit derivatives such as n^{th}-to-default swaps and CDOs, which have led to empirical contradictions that we address here. For example, Coval, Jurek, and Stafford [9] consider a *one-period* structural model, based on Merton [22] and incorporated within a CAPM framework, in order to price both index options and CDOs. The state prices are derived from options using an analog of Breeden and Litzenberger [4] in the presence of an implied volatility smile, while the model parameters are fitted to the 5-year CDX index spread. The authors conclude that the large skew from out-of-the-money put options leads to the overpricing of senior tranches, and hence, the catastrophe risk priced in the equity market is not fully accounted for in the credit market.

On the other hand, a few authors recently have argued that the equity index options market and multi-name credit derivatives market are indeed consistent. In particular, Collin-Dufresne,

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Goldstein, and Yang [8] use Black-Cox's *dynamic* structural model within CAPM to jointly price long-dated S&P options and CDO tranches. The authors match the entire term structure of CDX index spreads instead of just the 5-year spread in order to accurately capture information regarding the timing of expected defaults and the specification of idiosyncratic dynamics. Finally, they show that the pricing of CDO tranches is consistent with the pricing of equity index options, thus contradicting the findings reported by Coval, Jurek, and Stafford [9]. In support of Collin-Dufresne, Goldstein, and Yang [8], Luo and Carverhill [21] employ the three-factor portfolio intensity model of Longstaff and Rajan [19], which allows for firm-specific, industry, and economywide default events, and conclude that the CDO tranche market is well integrated with the S&P 500 index option market. Also, Li and Zhao [18] argue that the model of Coval, Jurek, and Stafford [9], which relies on out-of-sample analysis, is mis-specified because it is not flexible enough to even capture CDX tranche prices *in sample*. In addition, Li and Zhao use a skewed-t copula model to jointly price index options and CDO tranches, and they conclude that the CDO market is actually efficient.

In this paper, we consider a dynamic hybrid *intensity-based* model for valuing equity index options and multi-name credit derivatives. In the single-name case, Carr and Wu [6] discuss a joint equity-credit framework in which a stock has CIR stochastic volatility and jumps to default, while the intensity of default is an affine function of the stochastic variance process and an idiosyncratic process. In Carr and Linetsky [5], the stock price has constant elasticity of variance (CEV) and jumps to default. Meanwhile, Bayraktar [2] and Bayraktar and Yang [3] consider a jump-to-default model for the stock, where the intensity is a function of the volatility, which is driven by fast and slow mean-reverting processes.

We extend this hybrid formulation to the multi-name case. We assume that the intensity is an affine function of the stock index variance and an idiosyncratic (firm) component, following Carr and Wu [6] but where we model the stock market index rather than an individual stock. This modification allows us to price options on the stock index in a closed form because the index and its variance follow the stochastic volatility with jumps in variance (SVJ-V) model. In addition, we suppose that the market factor (i.e., the variance) and idiosyncratic processes are affine jump diffusions (AJDs), as in Mortensen [23]. Then, in this *bottom-up* approach, we can analytically price single-name credit derivatives such as CDSs and also efficiently price multi-name credit derivatives such as CDOs using recursion and Fourier inversion. The semi-analytical valuations allow us to calibrate our model to i) equity index options and CDSs and ii) CDOs and CDSs, and this allows us to assess the systematic risks inherent in equity and credit markets. Finally, we compare our results to related works such as Coval, Jurek, and Stafford [9] and Collin-Dufresne, Goldstein, and Yang [8], and we explain how our results differ from the existing literature.

Here is an outline of this paper. In Section 2, we introduce the bottom-up intensity-based common-factor model that we will use for pricing both credit derivatives and equity index options, and then we discuss the CDO framework, that is, the tranche structure, the payment streams, and the fair values. In Section 3, we follow up with the algorithms for valuing CDO tranche spreads, CDS spreads and equity index options under our model. Section 4 describes the sources of the market data and various market conventions. In Section 5, we assess the systematic risks in the equity and credit markets in two ways: first, we examine the forward problem where we calibrate our model parameters from equity index options and CDS spreads and then price CDO tranches; secondly, we examine the backward problem where we fit the model parameters to CDO tranche spreads and CDS spreads and then price equity index options. In Section 6, we provide empirical results for these two problems, and then we conclude the paper in Section 7 by analyzing the systematic risks in credit and equity markets.

2 Stochastic Model and CDO Framework

Let $(\Omega, \mathcal{F}, \mathbb{P}^*)$ be a probability space, where $\mathbb{P}^*$ is the risk-neutral probability measure reflected by market prices of credit derivatives. We consider a CDO on a portfolio of M underlying firms, labelled 1 to M.

2.1 Intensity-Based Common-Factor Model

For $i = 1, \ldots, M$, we let τ_i be the default time of the i^{th} firm and we let $(\lambda_t^i)_{t\geq 0}$ be the intensity process of firm i. As in Mortensen [23], we suppose that λ_t^i has the form

$$\lambda_t^i = X_t^i + c_i Y_t, \tag{2.1}$$

where $c_i > 0$ is a constant, $X^i = (X_t^i)_{t\geq 0}$ is the idiosyncratic process for the i^{th} firm, and $Y = (Y_t)_{t\geq 0}$ is a market factor process that is independent of the X^i but affects all of the firms. We let X^i and Y be affine jump diffusions (AJDs) that satisfy the stochastic differential equations (SDEs)

$$dX_t^i = \kappa_i(\bar{x}_i - X_t^i)dt + \sigma_i\sqrt{X_t^i}dW_t^i + dJ_t^i, \tag{2.2}$$

$$dY_t = \kappa_Y(\bar{y} - Y_t)dt + \sigma_Y\sqrt{Y_t}dW_t^Y + dJ_t^Y, \tag{2.3}$$

where

- $W^Y, W^1, \ldots, W^M$ are independent $\mathbb{P}^*$-Brownian motions,
- $J^Y, J^1, \ldots, J^M$ are independent compound Poisson processes under $\mathbb{P}^*$ with respective jump intensities $l_Y, l_1, \ldots, l_M$ and exponentially distributed jump sizes with respective means ξ_Y, $\xi_1, \ldots, \xi_M$,
- κ_i, $\bar{x}_i$, and σ_i are the respective mean-reversion speed, mean-reverting level, and volatility parameters for the process X^i,
- κ_Y, $\bar{y}$, and σ_Y are the respective mean-reversion speed, mean-reverting level, and volatility parameters for the process Y, and
- the Feller condition holds for the X^i and for Y:

$$\sigma_i^2 \leq 2\kappa_i\bar{x}_i, \qquad \sigma_Y^2 \leq 2\kappa_Y\bar{y},$$

  so that the processes X^i and Y will stay positive almost surely.

Note that (2.1)-(2.3) is a *bottom-up model* for credit derivative valuation, since we model the intensity for each underlying firm and then build up the loss distribution for the portfolio. Mortensen [23] observes that due to the jump components in X^i and Y, this model is able to capture the high correlation embedded in the senior CDO tranche spreads. Through an empirical analysis, Feldhütter and Nielsen [15] showed that this model is able to capture both the level and the time series dynamics of CDO tranche spreads.

In addition to the intensity process defined above, we incorporate the price process for the stock market index (e.g., the S&P 500 index), as denoted by $S = (S_t)_{t\geq 0}$. As motivated by Carr and Wu [6], we assume that S has a stochastic variance process which is a constant multiple of the market factor process Y:

$$dS_t = (r - q)S_t dt + \sqrt{b_Y Y_t}S_t dW_t^S, \tag{2.4}$$

where r is the risk-free interest rate, q is the continuous dividend yield, W^S is a $\mathbb{P}^*$-Brownian motion and b_Y is a positive constant. We suppose that the Brownian motions W^S and W^Y (from (2.3)) are correlated via

$$\mathbb{E}^*[dW_t^S \cdot dW_t^Y] = \rho\, dt,$$

where $\mathbb{E}^*$ represents the expectation operator under $\mathbb{P}^*$. Empirical evidence suggests that the market return process and its volatility are negatively correlated (this is termed the leverage effect); hence, we assume that $\rho < 0$. In addition, there is evidence that credit spreads are positively correlated to the market return volatilities; see, for example, Collin-Dufresne, Goldstein, and Martin [7]. We capture this relation through the positive coefficients c_i in (2.1) and b_Y in (2.4). The initial values of the processes X^i, Y, and S are given, respectively, by

$$X_0^i = x_0^i, \quad Y_0 = y_0, \quad S_0 = \bar{S}_0. \tag{2.5}$$

2.2 CDO Framework

Let us describe the framework of a collateralized debt obligation (CDO) by describing the tranche structure, the payment legs, and the fair value of the legs. For more details on the CDO framework, see Lando [17] or Elizalde [13].

We make the assumption that the total notional is 1 unit. We let $N = (N_t)_{t\geq 0}$ be the portfolio loss process, that is,

$$N_t = \sum_{i=1}^{M} 1_{\{\tau_i \leq t\}},$$

where τ_i denotes the default time of the i^{th} firm. Usually defaults do not incur full loss of the value of the underlying bonds, as there is some recovery. Suppose $\delta_r \in [0, 1)$ is the fraction recovered, so that $(1 - \delta_r)$ is the fractional loss incurred on each default, sometimes called the loss-given-default. Then, the *fractional* portfolio loss is

$$L_t = \frac{(1 - \delta_r)N_t}{M}. \tag{2.6}$$

2.2.1 CDO Tranches

A typical CDO consists of several tranches, such as *equity*, *mezzanine* and *senior*. Each tranche is characterized by an attachment (K_L) and detachment (K_U) point; an investor who buys this tranche is responsible for all losses occurring in the interval $[K_L, K_U]$. The equity tranche is the riskiest and requires the highest premium; the mezzanine tranches are the intermediate tranches with medium-level risk; the senior (or super senior) tranche is of the highest credit quality and requires the lowest premium.

Table 1 displays the attachment and detachment points for each of the 6 tranches, namely the *Equity*, *Mezzanine 1*, *Mezzanine 2*, *Mezzanine 3*, *Senior*, and *Super Senior* tranches, for the two standardized credit indices, the Dow Jones CDX (comprising North American companies) and the Dow Jones iTraxx (comprising European companies). In our analysis, we focus on the CDX tranches because the firms in the CDX appear to be more representative of the firms comprising the S&P 500 index, on which we compute option prices.

We now define the tranche loss process, which will be used to define the payment legs.

Definition 1. *For the tranche characterized by* $[K_L, K_U]$*, the* tranche loss *at time* t *is defined by* $F(L_t)$*, where* L_t *is the fractional portfolio loss at time* t*, given by* (2.6)*, and the function* F *is given by*

$$F(x) = (x - K_L)^+ - (x - K_U)^+, \quad x \in [0, 1].$$

Table 1: Attachment (K_L) and Detachment (K_U) Points for the Tranches of the Dow Jones CDX and Dow Jones iTraxx indices.

	Equity		**Mezz 1**		**Mezz 2**		**Mezz 3**		**Senior**		**SuperSen**	
	K_L	K_U	K_L	K_U	K_L	K_U	K_L	K_U	K_L	K_U	K_L	K_U
CDX	0%	3%	3%	7%	7%	10%	10%	15%	15%	30%	30%	100%
iTraxx	0%	3%	3%	6%	6%	9%	9%	12%	12%	22%	22%	100%

Observe that F is equivalent to the payoff of a call spread on the loss fraction, with "strikes" given by the attachment and detachment points.

2.2.2 Payment Legs

Each CDO tranche has two legs: the *premium* or fixed leg comprising an upfront payment along with periodic payments from the protection buyer to the tranche holder (i.e., the protection seller) on the remaining notional for the tranche; and the *protection* or floating leg where payments are made by the tranche holder to the protection buyer as the losses impact the tranche. The premium payments are made at the K regular payment dates $T_1, T_2, \ldots, T_K = T$, with

$$T_k = k\Delta\tau,\ k = 1, \ldots, K, \quad \text{and} \quad \Delta\tau = \frac{T}{K}.$$

We make the mild assumption that insurance payouts in the protection leg are also made at the same payment dates, covering all the losses since the previous payment date. To be precise, here are the payment streams:

(a) For the *premium leg*, the tranche holder receives from the protection buyer the following:

- an upfront payment of $U(K_U - K_L)$ at time 0, and
- $R(T_k - T_{k-1})[K_U - K_L - F(L_{T_k})]$ at each payment date $T_k,\ k = 1, 2, \ldots, K$,

where U is the upfront fee, R is the annualized running spread, and the amount $K_U - K_L - F(L_{T_k})$ is the remaining notional for the tranche at time T_k. Note that some tranches pay only the running spread, while other tranches pay the upfront fee in addition to the running spread; see Section 4.2 for more details.

(b) For the *protection leg*, the tranche holder pays the protection buyer

- $F(L_{T_k}) - F(L_{T_{k-1}})$ at each payment date $T_k,\ k = 1, 2, \ldots, K$.

Here, the difference $F(L_{T_k}) - F(L_{T_{k-1}})$ represents the loss incurred between the previous payment date T_{k-1} and the current payment date T_k as it impacts the tranche that the holder is responsible for.

2.2.3 Fair Value

Under the risk-neutral probability $\mathbb{P}^*$, the fair value of the premium leg is given by

$$Prem = U(K_U - K_L) + R\sum_{k=1}^{K}(T_k - T_{k-1})e^{-rT_k}\,\mathbb{E}^*\left[K_U - K_L - F(L_{T_k})\right], \tag{2.7}$$

and the fair value of the protection leg is given by

$$Prot = \sum_{k=1}^{K} e^{-rT_k}\, \mathbb{E}^* \left[F(L_{T_k}) - F(L_{T_{k-1}}) \right]. \tag{2.8}$$

Once we know the fair values of the premium and protection legs, we can compute the running spread or the upfront fee, depending on which one is given in the market.

Definition 2. *(i) When the upfront fee U is given, the* running spread *is the number R that equates the fair values of the two legs* (2.7) *and* (2.8) *and is hence given by*

$$R = \frac{\sum_{k=1}^{K} e^{-rT_k}\, \mathbb{E}^*[F(L_{T_k}) - F(L_{T_{k-1}})] - U(K_U - K_L)}{\sum_{k=1}^{K} (T_k - T_{k-1}) e^{-rT_k}\, \mathbb{E}^*[K_U - K_L - F(L_{T_k})]}. \tag{2.9}$$

(ii) On the other hand, when the running spread R is given, the upfront fee *is the number U that equates the fair values of the two legs* (2.7) *and* (2.8) *and is hence given by*

$$\begin{aligned} U = \frac{1}{K_U - K_L} \Bigg[& \sum_{k=1}^{K} e^{-rT_k}\, \mathbb{E}^*[F(L_{T_k}) - F(L_{T_{k-1}})] \\ & - R \sum_{k=1}^{K} (T_k - T_{k-1}) e^{-rT_k}\, \mathbb{E}^*[K_U - K_L - F(L_{T_k})] \Bigg]. \end{aligned} \tag{2.10}$$

To compute the expectations $\mathbb{E}^*[F(L_{T_k})]$, $k = 1, \ldots, K$, in (2.9) and (2.10), we need to determine the *portfolio loss distribution*, as explained below in Section 3.1.

3 Product Valuation

In this section, we discuss the valuation of CDOs, CDSs, and equity index options under our hybrid intensity model.

3.1 Intensity-Based Valuation of CDOs

In this section, we describe our algorithm for valuing CDO tranches under the intensity-based common factor model of Section 2. Recall that τ_i is the default time of the i^{th} firm. Let $U = (U_t)_{t \geq 0}$ be the integrated market factor process, defined by

$$U_t = \int_0^t Y_s ds, \quad t \geq 0,$$

where Y satisfies the SDE (2.3).

3.1.1 Single-Name Conditional Default Probabilities

Given the integrated market factor $U_t = u$, the conditional default probability for the i^{th} firm is equal to

$$\begin{aligned} p_i(t|u) &:= \mathbb{P}^*\{\tau_i \leq t \mid U_t = u\} \\ &= 1 - \mathbb{E}^*[e^{-\int_0^t \lambda_s^i ds} \mid U_t = u] \\ &= 1 - e^{-c_i u} \mathbb{E}^*[e^{-\int_0^t X_s^i ds}] \quad \text{since } \lambda_s^i = X_s^i + c_i Y_s \text{ with } X^i \perp\!\!\!\perp Y \\ &= 1 - e^{-c_i u} e^{\alpha_i(t) + \beta_i(t) x_0^i}, \end{aligned} \tag{3.1}$$

where the functions $\alpha_i(\cdot)$ and $\beta_i(\cdot)$ are given explicitly by Equation (A.4) in Appendix A.1, using $q = -1$ and the suitable parameters for the process X^i.

3.1.2 Conditional Loss Distribution

Let us determine the conditional loss distribution for the portfolio. As the CDO contains a heterogeneous pool of firms, we use the recursive algorithm suggested by Andersen, Sidenius, and Basu [1] and employed by Mortensen [23]. With $t > 0$ fixed, we define $P_k(m|u)$ as the probability of exactly m defaults by time t among the first k entities, conditional on the integrated market factor taking the value u. Explicitly, we have

$$P_k(m|u) = \mathbb{P}^*\{N_t^k = m \mid U_t = u\},$$

where N_t^k denotes the number of defaults by time t among the first k entities. For the recursion, we start from $P_0(m|u) = 1_{\{m=0\}}$, $m = 0, \ldots, M$, and then we use conditional independence to obtain the following recurrence relation, for $k = 1, \ldots, M$:

$$\begin{aligned} P_k(m|u) &= P_{k-1}(m|u)(1 - p_k(t|u)) + P_{k-1}(m-1|u)p_k(t|u), \quad m = 1, \ldots, k, \\ P_k(0|u) &= P_{k-1}(0|u)(1 - p_k(t|u)), \end{aligned} \tag{3.2}$$

with the convention $P_k(m|u) = 0$ for $m > k$, and where $p_k(t|u)$ is given by (3.1). Finally, the conditional loss distribution for the total portfolio, given $U_t = u$, is

$$\mathbb{P}^*\{N_t = m \mid U_t = u\} = \mathbb{P}^*\{N_t^M = m \mid U_t = u\} = P_M(m|u), \quad m = 0, 1, \ldots, M. \tag{3.3}$$

3.1.3 Density of Integrated Market Factor

Let us determine the density f_{U_t} of the integrated market factor $U_t = \int_0^t Y_s ds$ by fast Fourier transform (FFT) methods. To that end, the characteristic function of U_t is defined by

$$\Phi_{U_t}(r) := \mathbb{E}^*[e^{irU_t}] = \int_0^\infty e^{iru} f_{U_t}(u) du, \quad r \in \mathbb{R}, \tag{3.4}$$

which is known in closed form via the solution of Riccati ODEs, as described in Appendix A.2. By Fourier inversion, the density is given by

$$f_{U_t}(u) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{-iur} \Phi_{U_t}(r) dr, \quad u > 0.$$

Now, for some fixed integer N_U, we employ the Fourier grid points $r_j = (j-1)\Delta r,\ j = 1, \ldots, N_U$, and $u_k = (k-1)\Delta u,\ k = 1, 2, \ldots, N_U$, to obtain the approximation

$$f_{U_t}(u_k) \approx \frac{1}{\pi} \sum_{j=1}^{N_U} \mathrm{Re}\left(e^{-i\Delta r \Delta u (j-1)(k-1)} \Phi_{U_t}(\Delta r(j-1))\Delta r\right).$$

Since the discrete Fourier transform (DFT) requires that $\Delta r \cdot \Delta u = \frac{2\pi}{N_U}$, we can re-write the density as

$$f_{U_t}(u_k) \approx \sum_{j=1}^{N_U} \mathrm{Re}\left(e^{-i\frac{2\pi}{N_U}(j-1)(k-1)} \frac{\Delta r}{\pi} \Phi_{U_t}(\Delta r(j-1))\right), \quad k = 1, \ldots, N_U. \tag{3.5}$$

Since we are only concerned with the real part of the summation in (3.5), the approximation relies on finding a suitable value $\bar{r}$ such that $\mathrm{Re}(\Phi_{U_t}(r)) \approx 0$ for $r > \bar{r}$; then, from the boundary condition, we set $\Delta r = \bar{r}/(N_U - 1)$.

3.1.4 Portfolio Loss Distribution

The portfolio loss distribution is computed by integrating the conditional probability over the range of the integrated market factor. In particular, we have

$$\mathbb{P}^*\{N_t = k\} = \int_0^\infty \mathbb{P}^*\{N_t = k \mid U(t) = u\} \, f_{U_t}(u) du, \quad k = 0, \ldots, M, \tag{3.6}$$

where the conditional probability $\mathbb{P}^*\{N_t = k \mid U_t = u\}$ is given by (3.3) and the density f_{U_t} is given by (3.5).

3.1.5 CDO Tranche Spread

Once we have computed the loss distribution $\mathbb{P}^*\{N_t = k\}$, $k = 0, \ldots, M$, from (3.6), we can calculate the expected tranche loss $\mathbb{E}^*[F(L_t)]$ at time t using the relation (2.6) and a simple weighted average:

$$\mathbb{E}^*[F(L_t)] = \sum_{k=0}^{M} F\left(\frac{(1-\delta_r)k}{M}\right) \mathbb{P}^*\{N_t = k\}. \tag{3.7}$$

Finally, to obtain the running spread R in (2.9) and the upfront fee U in (2.10), we substitute the expected tranche loss (3.7) at the payment dates $T_1, \ldots, T_K$. At time $T_0 = 0$, the expected tranche loss is simply $\mathbb{E}^*[F(L_0)] = \mathbb{E}^*[F(0)] = 0$ since $N_0 = 0$.

3.2 Credit Default Swap Valuation

For a brief description of a credit default swap (CDS) and its risk-neutral valuation, see Appendix B of Mortensen [23]. For a more detailed description of CDSs, see, for example, Duffie [10].

The *CDS spread* is given by the value $\bar{S}_i$ that equates the premium and protection legs and is given by the closed-form expression

$$\begin{aligned}\bar{S}_i = (1-\delta_r) \sum_{k=1}^{\bar{K}} e^{-r(\bar{T}_k + \bar{T}_{k-1})/2} \Big[\mathbb{P}^*\{\tau_i > \bar{T}_{k-1}\} - \mathbb{P}^*\{\tau_i > \bar{T}_k\}\Big] \Big/ \\ \sum_{k=1}^{\bar{K}} \left((\bar{T}_k - \bar{T}_{k-1}) e^{-r\bar{T}_k} \mathbb{P}^*\{\tau_i > \bar{T}_k\} + \frac{\bar{T}_k - \bar{T}_{k-1}}{2} e^{-r(\bar{T}_k + \bar{T}_{k-1})/2} \Big[\mathbb{P}^*\{\tau_i > \bar{T}_{k-1}\} - \mathbb{P}^*\{\tau_i > \bar{T}_k\}\Big] \right),\end{aligned} \tag{3.8}$$

where $\bar{T}$ is the maturity of the CDS (with notional of one dollar), $\bar{T}_1, \ldots, \bar{T}_{\bar{K}} = \bar{T}$ are the payment dates, and δ_r is the constant recovery rate.

3.3 Pricing of Equity Index Options

Recall that from the SDEs (2.3) and (2.4), the stock index (S_t) and the market factor (Y_t) satisfy the respective SDEs

$$\begin{aligned} dS_t &= (r-q)S_t dt + \sqrt{b_Y Y_t} S_t dW_t^S, \\ dY_t &= \kappa_Y(\bar{y} - Y_t)dt + \sigma_Y \sqrt{Y_t} dW_t^Y + dJ_t^Y, \end{aligned}$$

where W^S and W^Y are correlated Brownian motions with $\mathbb{E}^*[dW_t^S \cdot dW_t^Y] = \rho\, dt$, and J^Y is a compound Poisson process with jump intensity l_Y and exponentially distributed jump sizes with mean ξ_Y. Noting that $Z_t := b_Y Y_t$, $t \geq 0$, defines the variance process for (S_t), we have the stochastic volatility with jumps in volatility (SVJ-V) model, as introduced in Section 4 of Duffie, Pan, and Singleton [12], that is,

$$
\begin{aligned}
dS_t &= (r-q)S_t dt + \sqrt{Z_t} S_t dW_t^S, \\
dZ_t &= \kappa_Z(\bar{z} - Z_t)dt + \sigma_Z \sqrt{Z_t} dW_t^Z + dJ_t^Z,
\end{aligned}
\tag{3.9}
$$

where $W^Z = W^Y$, J^Z is a compound Poisson process with intensity l_Z and exponentially distributed jump sizes with mean ξ_Z, and the parameters for Z are given by $\kappa_Z = \kappa_Y$, $\sigma_Z = \sqrt{b_Y}\sigma_Y$, $\bar{z} = b_Y \bar{y}$, $z_0 = b_Y y_0$, $l_Z = l_Y$, and $\xi_Z = b_Y \xi_Y$. Hence, the variance parameters in the model are given by

$$
\chi = \{\kappa_Z, \sigma_Z, \bar{z}, z_0, l_Z, \xi_Z, \rho\}. \tag{3.10}
$$

In the proposition below, we give the price of a call option on the stock index under the SVJ-V model. First, we define the relevant variables for pricing. In particular, we let

$$
\bar{\beta}(\tau, u) = -\frac{\bar{a}(u)(1 - e^{-\gamma(u)\tau})}{2\gamma(u) - (\gamma(u) + \bar{b}(u))(1 - e^{-\gamma(u)\tau})}. \tag{3.11}
$$

where $\bar{b}$, $\bar{a}$, and γ are given, respectively, by

$$
\begin{aligned}
\bar{b}(u) &= \sigma_Z \rho u - \kappa_Z, \\
\bar{a}(u) &= u(1-u),
\end{aligned}
$$

and

$$
\gamma(u) = \sqrt{\bar{b}(u)^2 + \bar{a}(u)\sigma_Z^2}.
$$

Also, we define

$$
\bar{\alpha}(\tau, u) = \alpha_0(\tau, u) - l_Z \tau + l_Z \int_0^\tau \theta(u, \bar{\beta}(s, u)) ds, \tag{3.12}
$$

where α_0 is given by

$$
\alpha_0(\tau, u) = -r\tau + (r-q)u\tau - \kappa_Z \bar{z} \left(\frac{\gamma(u) + \bar{b}(u)}{\sigma_Z^2} \tau + \frac{2}{\sigma_Z^2} \log\left[1 - \frac{\gamma(u) + \bar{b}(u)}{2\gamma(u)} (1 - e^{-\gamma(u)\tau}) \right] \right),
$$

and θ is the transform of the jump-size distribution ν for (S_t, Z_t):

$$
\theta(c_1, c_2) = \int_{\mathbb{R}^2} e^{c_1 z_1 + c_2 z_2} d\nu(z_1, z_2), \quad c_1, c_2 \in \mathbb{C}.
$$

Remark 1. *In this SVJ-V model, the first component (i.e., the stock process) does not have jumps, while the second component (i.e., the variance process) has an exponential jump distribution with mean ξ_Z. Hence, the jump-size distribution ν is identified by*

$$
d\nu(z_1, z_2) = \delta_0(dz_1) 1_{(0,\infty)}(z_2) \frac{1}{\xi_Z} e^{-\frac{z_2}{\xi_Z}} dz_2, \quad z_1, z_2 \in \mathbb{R},
$$

and the transform of ν is given by

$$
\theta(c_1, c_2) = \int_{\mathbb{R}_+} e^{c_2 z_2} \frac{1}{\xi_Z} e^{-\frac{z_2}{\xi_Z}} dz_2 = \frac{1}{1 - c_2 \xi_Z}, \quad \textit{when } \operatorname{Re}(c_2) < \frac{1}{\xi_Z}.
$$

Integrating $\theta(u, \bar{\beta}(s, u))$ with respect to s, we obtain the integral in the expression for $\bar{\alpha}$ above:

$$
\begin{aligned}
\int_0^\tau \theta(u, \bar{\beta}(s, u)) ds &= \int_0^\tau \frac{1}{1 - \bar{\beta}(s, u)\xi_Z} ds = \int_0^\tau \frac{2\gamma(u) - (\gamma(u) + \bar{b}(u))(1 - e^{-\gamma(u)s})}{2\gamma(u) + [\xi_Z \bar{a}(u) - (\gamma(u) + \bar{b}(u))](1 - e^{-\gamma(u)s})} ds \\
&= \frac{\gamma(u) - \bar{b}(u)}{\gamma(u) - \bar{b}(u) + \xi_Z \bar{a}(u)} \tau - \frac{2\xi_Z \bar{a}(u)}{\gamma(u)^2 - (\bar{b}(u) - \xi_Z \bar{a}(u))^2} \log\left(1 - \frac{(\gamma(u) + \bar{b}(u)) - \xi_Z \bar{a}(u)}{2\gamma(u)} (1 - e^{-\gamma(u)\tau}) \right).
\end{aligned}
$$

Proposition 1. *Let* (S_t, Z_t) *be the stock index process and variance process satisfying the SVJ-V model in* (3.9)*, and let* χ *be the set of variance parameters* (3.10)*. Denote* ψ^χ *as the discounted transform of the log-state price* $s_t = \log S_t$*:*

$$\psi^\chi(u,(s,z),t,T) = \mathbb{E}^{*,\chi}\left[e^{-r(T-t)}e^{us_T} \mid s_t = s, Z_t = z\right].$$

Then, the price C *at time* 0 *of a European call option with strike* K *and maturity* T *can be written as*

$$\boxed{\begin{aligned} C(0,K,T,(\bar{S}_0,z_0),\chi) &= \mathbb{E}^{*,\chi}\left[e^{-rT}(S_T-K)^+ \mid S_0 = \bar{S}_0, Z_0 = z_0\right] \\ &= G_{1,-1}(-\log K;(\log\bar{S}_0,z_0),T,\chi) - KG_{0,-1}(-\log K;(\log\bar{S}_0,z_0),T,\chi) \end{aligned}}, \tag{3.13}$$

where $G_{a,b}$*, for* $a, b \in \mathbb{R}$*, is given by*

$$G_{a,b}(w;(s,z),T,\chi) = \frac{\psi^\chi(a,(s,z),0,T)}{2} - \frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im}\left(\psi^\chi(a+i\xi b,(s,z),0,T)e^{-i\xi w}\right)}{\xi}d\xi.$$

Here, $\operatorname{Im}(c)$ *denotes the imaginary part of* $c \in \mathbb{C}$*, and* ψ^χ *simplifies to*

$$\psi^\chi(u,(s,z),t,T) = \exp\left(\bar{\alpha}(T-t,u) + us + \bar{\beta}(T-t,u)z,\right),$$

with $\bar{\alpha}$ *and* $\bar{\beta}$ *defined by* (3.12) *and* (3.11)*, respectively. In addition, the price* P *of a European put option can be obtained from the put-call parity relation,*

$$P = C - \bar{S}_0e^{-qT} + Ke^{-rT}.$$

Proof. Here is an outline of the proof. First, the form of C in (3.13) arises from the payoff of a standard call option; see, for example, Heston [16]. Then, the expression of $G_{a,b}$ arises from Fourier inversion as stated in Proposition 2 (and proven in Appendix A) of Duffie, Pan, and Singleton [12]. Finally, the reduced form of ψ^χ is provided in Section 4 of Duffie, Pan, and Singleton [12]. □

4 Market Data and Conventions

In this section, we discuss the source of the market data, the CDX Series information, and the CDS conventions and adjustments that will be used in our empirical testing of the model.

4.1 Data

Our primary data are equity index options, CDS spreads, and CDO tranche spreads. In particular, we consider European put options on the S&P 500 index (SPX), CDS spreads based on the underlying firms of the CDX North American Investment Grade Index (CDX.NA.IG), and CDO tranche spreads written on the CDX.NA.IG. We obtained the SPX option data from OptionMetrics via Wharton Research Data Services (WRDS), and we retrieved the CDS and CDO tranche spreads from Bloomberg.

To compare risks across equity and credit markets, it seems plausible to examine instruments with similar maturities. However, we note that the credit instruments in the market, in general, had longer maturities than the equity index options. In this case, we selected the SPX options with the longest maturities of just over two years, we considered CDSs with maturities of 1 and 5 years, and we examined CDOs with maturities of 5 years.

4.2 CDX Series Information

In this subsection, we provide general information for the CDX.NA.IG series, which rolls over every 6 months, on March 20 and September 20 of each year, with a possible one- or two-day settlement lag. For Series 1 to 11, that is, up until early 2009, only the equity tranche (0-3%) paid an upfront fee with a running spread of $R = 500$ bps, while the other tranches had no upfront fee. We note, however, that these conventions changed for *off-the-run* (i.e., not current) series beginning in mid-2009. In particular, as of June 20, 2009, for Series 11 and earlier, the equity (0-3%), mezzanine 1 (3-7%), and mezzanine 2 (7-10%) tranche spreads are now quoted as upfront fees with 500-bp coupons, while the mezzanine 3 (10-15%), senior (15-30%), and super senior (30-100%) tranche spreads in the market are quoted as upfront fees with 100-bp coupons. For Series 12 to 14, coinciding with the tail end of the credit crisis, we note that all of the tranches paid upfront fees but had a lower running spread of $R = 100$ bps. Lastly, we note that the tranche structure was altered for Series 15, which rolled in September 2010, and for all subsequent series. In particular, for these newer series, the 4 tranches are 0-3%, 3-7%, 7-15%, and 15-100%, and the market spreads are quoted as upfront fees with respective running spreads of 500 bps, 100 bps, 100 bps, and 25 bps.

In our calibration, we ignore the super senior tranche (corresponding to the 30-100% tranche for Series 1 to 14 and the 15-100% tranche for Series 15 and beyond) of the CDO as it is very illiquid and difficult to fit to an intensity model. Note that Mortensen [23], Papageorgiou and Sircar [24], Collin-Dufresne, Goldstein, and Yang [8], and others also do not consider the super senior tranche. In particular, Collin-Dufresne, Goldstein, and Yang [8] argue that calibrating market dynamics to match the super senior tranche impacts equity state prices significantly only for moneyness levels around 0.2 or lower (corresponding to very illiquid products); this indicates that one cannot 'extrapolate' the information in option prices to deduce information regarding the super senior tranche.

4.3 CDS Conventions and Adjustments

The maturity of the credit default swaps fall on the International Monetary Market (IMM) Dates, in particular, March 20, June 20, September 20 and December 20 of each calendar year. Thus, for example, a 'five-year' contract traded any time between September 21, 2005 and December 20, 2005 would have termination date of December 20, 2010, implying that the actual maturity would be more than 5 years.

The *CDX spread* is the coupon for the CDS index and is equivalent to the running spread for the [0%, 100%] tranche of the CDX. To avoid a theoretical arbitrage, the CDX spread should equal the average of the CDS spreads, but in practice, this is not the case; the difference is termed the CDS-CDX basis.

Table 2 below shows the CDX spreads compared with the average CDS spreads from the market for 15 selected dates from 2004 to 2010 that we will use for our testing. We observe that for all of the dates since March 2008, the average CDS spreads are indeed larger than the CDX spreads observed in the market.

To correct this basis on each date, we first scale all of the 5-year CDS spreads in the market by a constant factor so that the average of the new 5-year spreads matches the 5-year CDX spread. Then, we multiply the 1-year CDS spreads in the market by the same factor (i.e., the ratio of the new 5-year CDS spreads to the old 5-year CDS spreads) in order to keep the ratios between

Date	Series	CDX	Avg. CDS
Aug. 23, 2004	Series 2	70	69.44
Dec. 5, 2005	Series 5	49	45.94
Oct. 31, 2006	Series 7	34	36.75
Jul. 17, 2007	Series 8	45	41.10
Mar. 14, 2008	Series 9	182	191.71
Jun. 20, 2008	Series 10	115	157.07
Oct. 16, 2008	Series 11	173	279.03
Jan. 8, 2009	Series 9	220	306.66
Apr. 8, 2009	Series 9	258	353.28
Jul. 8, 2009	Series 12	139	173.65
Sep. 8, 2009	Series 12	121	160.15
Dec. 8, 2009	Series 13	98	104.33
Mar. 8, 2010	Series 13	89	92.28
Jun. 8, 2010	Series 9	141	168.32
Sep. 8, 2010	Series 9	118	140.56

Table 2: 5-year CDX and Average 5-year CDS Spreads (bps)

the 1-year and 5-year CDS spreads unchanged. In particular, we set, for $i = 1, \ldots, M$,

$$\begin{aligned} \bar{S}^i_{new}(5) &= \bar{S}^i_{market}(5) \cdot \frac{\bar{S}_{CDX}(5)}{\frac{1}{M}\sum_{i=1}^M \bar{S}^i_{market}(5)}, \\ \bar{S}^i_{new}(1) &= \bar{S}^i_{market}(1) \cdot \frac{\bar{S}^i_{new}(5)}{\bar{S}^i_{market}(5)} = \bar{S}^i_{market}(1) \cdot \frac{\bar{S}_{CDX}(5)}{\frac{1}{M}\sum_{i=1}^M \bar{S}^i_{market}(5)}. \end{aligned} \tag{4.1}$$

We note that these adjustments are similar to those done by Feldhütter [14], who points out the maturity mismatch between the CDO and the underlying CDS contracts and then adjusts all the CDS spreads for a particular maturity by a constant factor such that the average CDS spread matches the CDX level reported by Markit.

5 Assessing Systematic Risks: Forward and Backward Problems

In this section, we describe the "forward" and "backward" calibration problems, which allow us to analyze the systematic risks inherent in credit and equity markets. First, in Section 5.1 we consider the forward problem, in which we start from equity index options, fit the CDS spreads and then compare the CDO tranche spreads from the model and the market. Then, in Section 5.2, we consider the backward problem, in which we fit the model parameters to the CDO tranche spreads and CDS spreads and then compare the equity index option volatilities from the model and the market. Subsequently, in Section 6, we present relevant empirical results for the calibration procedures in both the forward and backward directions.

5.1 Forward Problem: From Equity Index Options to CDO Tranche Spreads

In this section, we consider the forward problem, going from equity index options to CDO tranche spreads. In particular, we first calibrate the market factor parameters to the S&P 500 option data.

Then, using the market parameters as input, we calibrate the idiosyncratic parameters to the CDS spread data. Next, using the calibrated market and idiosyncratic parameters, we compute the CDO tranche spreads from the intensity-based model, and we compare the model spreads to the market-observed spreads. This idea is illustrated in Figure 1 below.

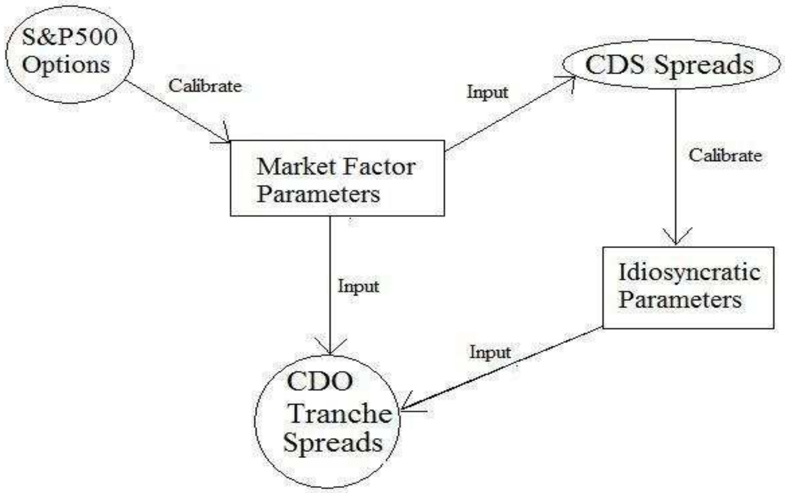

Figure 1: Forward Problem: From Equity Index Options to CDO Tranche Spreads

5.1.1 Calibration of Market Factor

We wish to calibrate the market factor parameters,

$$\chi_Y = \{\kappa_Y, \sigma_Y, \bar{y}, y_0, l_Y, \xi_Y, \rho, b_Y\}, \tag{5.1}$$

to the implied volatilities of S&P 500 options, as motivated by Coval, Jurek, and Stafford [9].

From Proposition 1 in Section 3.3 above, we know the analytical call price C under the SVJ-V model and hence, we can uniquely determine the Black-Scholes implied volatility, $\sigma_{imp\_model}(\chi_Y)$, for any set of market factor parameters χ_Y. We also know the implied volatilities of the S&P 500 options from the market, say $\sigma^j_{imp\_market}$, $j = 1\ldots,\bar{J}$, for some integer $\bar{J}$. Then, to calibrate the market factor parameters χ_Y, we run an optimization scheme to minimize the sum of squared errors (SSE),

$$SSE = \sum_{j=1}^{\bar{J}} \left(\sigma^j_{imp\_model}(\chi_Y) - \sigma^j_{imp\_market}\right)^2. \tag{5.2}$$

This can be implemented, for example, by using the *lsqnonlin* function in MATLAB.

5.1.2 Calibration of Idiosyncratic Components

For $i = 1,\ldots,M$, we wish to calibrate the idiosyncratic parameters,

$$\eta_i = \{\kappa_i, \sigma_i, \bar{x}_i, x^i_0, l_i, \xi_i, c_i\}, \tag{5.3}$$

to the adjusted market CDS spreads for the i^{th} firm, where we recall that the market spreads are adjusted according to Equation (4.1) in Section 4.3. Here is our calibration procedure for the i^{th} firm, $i = 1,\ldots,M$.

1. We first define $\bar{w} \in [0, 1]$ as a correlation parameter that represents the systematic share of the intensities, and hence, $1 - \bar{w}$ represents the idiosyncratic share of the intensities. This correlation parameter can be chosen freely; for example, we can set $\bar{w}$ to be the calibrated value from the backward problem in Section 5.2.

2. We make the following parsimonious assumptions for the idiosyncratic parameters:

$$\kappa_i = \kappa_Y, \quad l_i = l_Y \frac{1 - \bar{w}}{\bar{w}},$$

$$c_i = \frac{\bar{w}\bar{x}_i}{(1 - \bar{w})\bar{y}}, \quad \sigma_i = \sqrt{c_i}\sigma_Y, \quad \xi_i = c_i \xi_Y,$$

where the market factor parameters κ_Y, σ_Y, $\bar{y}$, l_Y, and ξ_Y were calibrated above in Section 5.1.1. (As for the rest of the market factor parameters in the set χ_Y, y_0 is an input for the CDS pricing below while ρ and b_Y are not used in the calibration of the idiosyncratic components.) From these assumptions for the idiosyncratic parameters, we observe that κ_i and l_i are fixed; c_i, σ_i, and ξ_i are dependent on $\bar{x}_i$; and $\bar{x}_i$ and x_0^i are to be explicitly calibrated.

3. We calibrate $\{\bar{x}_i, x_0^i\}$ to the 1-year and 5-year CDS spreads from the market, while enforcing the above conditions for the idiosyncratic parameters κ_i, l_i, c_i, σ_i, and ξ_i.

   We now provide more precise details of the calibration. We let $\bar{S}^{i,j}_{market}$ and $\bar{S}^{i,j}_{model}(\chi_Y, \eta_i)$ be the respective market and model CDS spreads for maturity T_j, with $T_1 = 1$ and $T_2 = 5$. Here, the model spread is computed from the closed-form expression (3.8), using the set of market factor parameters χ_Y and the set of idiosyncratic parameters η_i. Then, for the calibration, we minimize the sum of squared differences between the market and model-implied CDS spreads, that is,

$$\sum_{j=1}^{2} \left(\bar{S}^{i,j}_{model}(\chi_Y, \eta_i) - \bar{S}^{i,j}_{market} \right)^2.$$

   This optimization can be implemented, for example, by using the *lsqnonlin* function in MATLAB.

5.1.3 Market vs. Model CDO Tranche Spreads

Once we have calibrated the market factor parameters χ_Y and the idiosyncratic parameters η_i, for $i = 1, \ldots, M$, we price the CDO tranche spreads in our model by following the algorithm in Section 3.1. Then, by comparing the model and market tranche spreads, we analyze the systematic risks in equity and credit markets. In particular, since the model is calibrated to equity index options, we observe that if the senior spreads in the market are larger (smaller) than those from the model, then this indicates that the systematic risk in the credit market is greater (smaller) than that in the equity market.

5.2 Backward Problem: From CDO Tranche Spreads to Equity Index Options

In this section, we analyze the systematic risks inherent in credit and equity markets by considering the backward problem, going from CDO tranche spreads to equity index options. In particular, we first calibrate the market factor and idiosyncratic parameters to both the CDO tranche spreads and the CDS spreads from the market. Then, using the calibrated parameters, we compute the equity index option prices from the SVJ-V model and obtain the corresponding Black-Scholes implied volatilities. Finally, we compare the model implied volatilities to the market implied volatilities, and by examining the volatility skews, we measure the systematic tail risks inherent in credit and equity markets. This idea is illustrated in Figure 2 below.

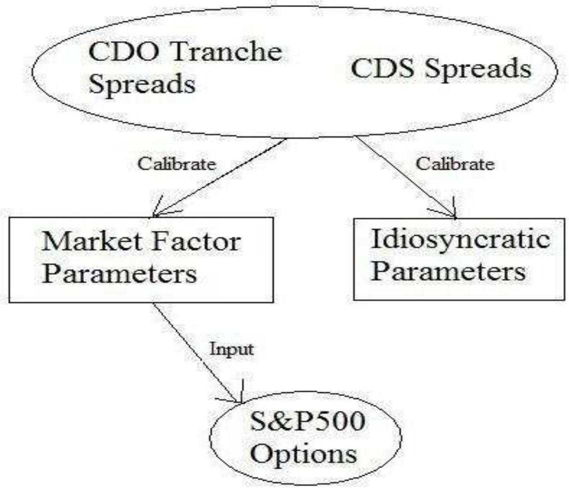

Figure 2: Backward Problem: From CDO Tranche Spreads to Equity Index Options

5.2.1 Fitting Parameters to CDO Tranches and CDS Spreads

We wish to fit the market factor parameters χ_Y in (5.1) and the idiosyncratic parameters η_i in (5.3), for $i = 1, \ldots, M$, to both the CDO tranche spreads and CDS spreads. Note that the market factor parameters ρ and b_Y do not impact the pricing of the credit derivatives and hence are free to be chosen later on. Let us explain how we calibrate the rest of the market factor parameters,

$$\eta_Y := \{\kappa_Y, \sigma_Y, \bar{y}, y_0, l_Y, \xi_Y\},$$

along with the idiosyncratic parameters

$$\eta_i = \{\kappa_i, \sigma_i, \bar{x}_i, x_0^i, l_i, \xi_i, c_i\}.$$

Due to the large number of parameters $(6 + 7M)$ that need to be calibrated, we use a parsimonious version of the model that is both practical and flexible. In particular, we follow Section 4.2 of Mortensen [23] and we minimize the deviations from the market CDO tranche spreads while matching the individual credit curves.

5.2.2 Market vs. Model Implied Volatilities

Finally, once we have calibrated the model parameters η_Y and η_i, $i = 1, \ldots, M$, to the CDO tranche spreads and CDS spreads from the market, we can compare the Black-Scholes implied volatility skews from the market to those from the fitted model.

Let us now describe our procedure. For the market, we select put options of the longest maturity, that is, approximately two years, for the 15 selected dates shown in Table 2 in Section 4.3. Meanwhile, for the model, we use the fitted parameters η_Y from Section 5.2.1 and we choose a range of values for the free parameters b_Y and ρ (in the set χ_Y from (5.1)). We note that under this SVJ-V model, as specified in Section 3.3, b_Y determines the *level* of the Black-Scholes implied volatility curve, while ρ determines the *skew/smile* of the curve. Hence, we select b_Y to match the level of the curve, and then we vary ρ to try to match the skew. In this backward problem, we observe that if the model skew is greater (less) than the market skew for a wide range of ρ, then this indicates that the risk in the credit market is greater (less) than that in the equity market.

6 Empirical Quantification of Systematic Risks

Here we present the empirical results for both the forward problem and the backward problem, based on the 15 selected dates in Table 2.. Throughout this section, we use the following set of parameters for the CDO:

$$M = 125;\ r = 0.04;\ \delta_r = 0.35;\ T = 5;\ \Delta\tau = T_k - T_{k-1} = 0.25,\ k = 1, \ldots, K;$$

and for the CDS, we suppose that $\bar{T} \in \{1, 5\}$ and $\bar{T}_k - \bar{T}_{k-1} = 0.25,\ k = 1, \ldots, \bar{K}$.

6.1 Forward Problem

In this section, we present the numerical results for the forward problem, starting from the equity index options, calibrating to the CDS spreads, and comparing the values for the CDO tranche spreads.

6.1.1 Implied Volatility

Figures 3 and 4 display the market implied volatility and the model calibrated volatility for the 15 dates above. For each date, we considered the options with maturity closest to two years.

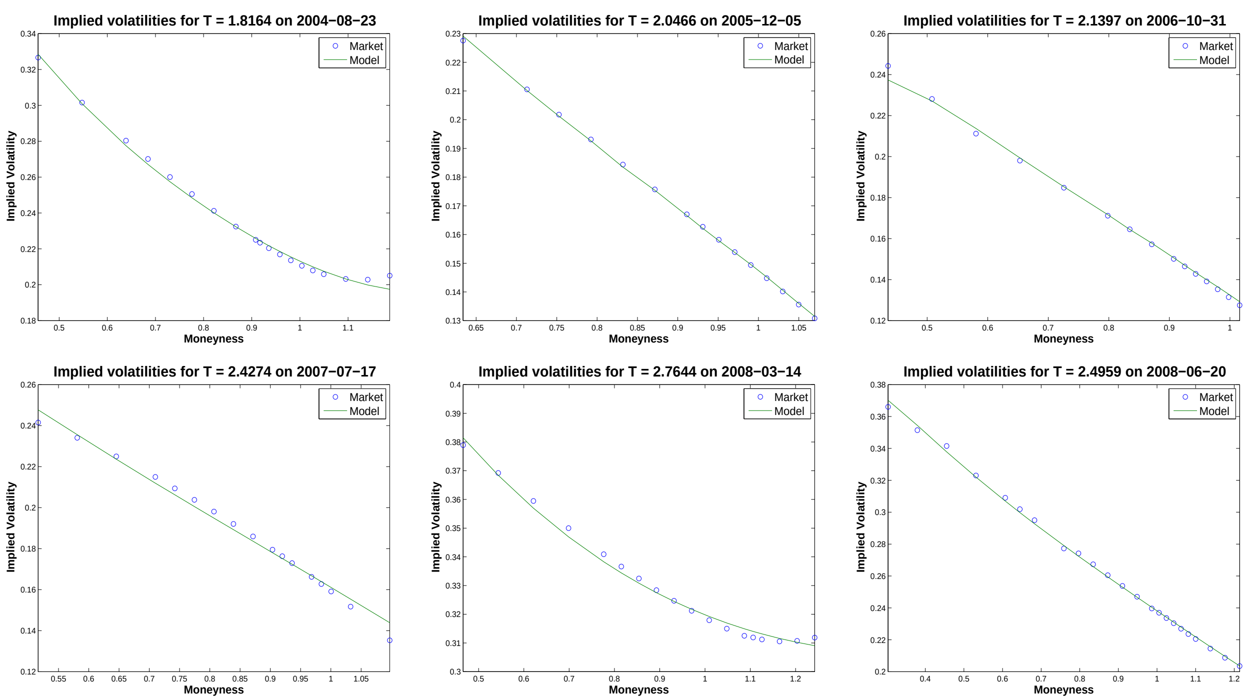

Figure 3: Implied Volatility, Aug. 23, 2004 to June 20, 2008

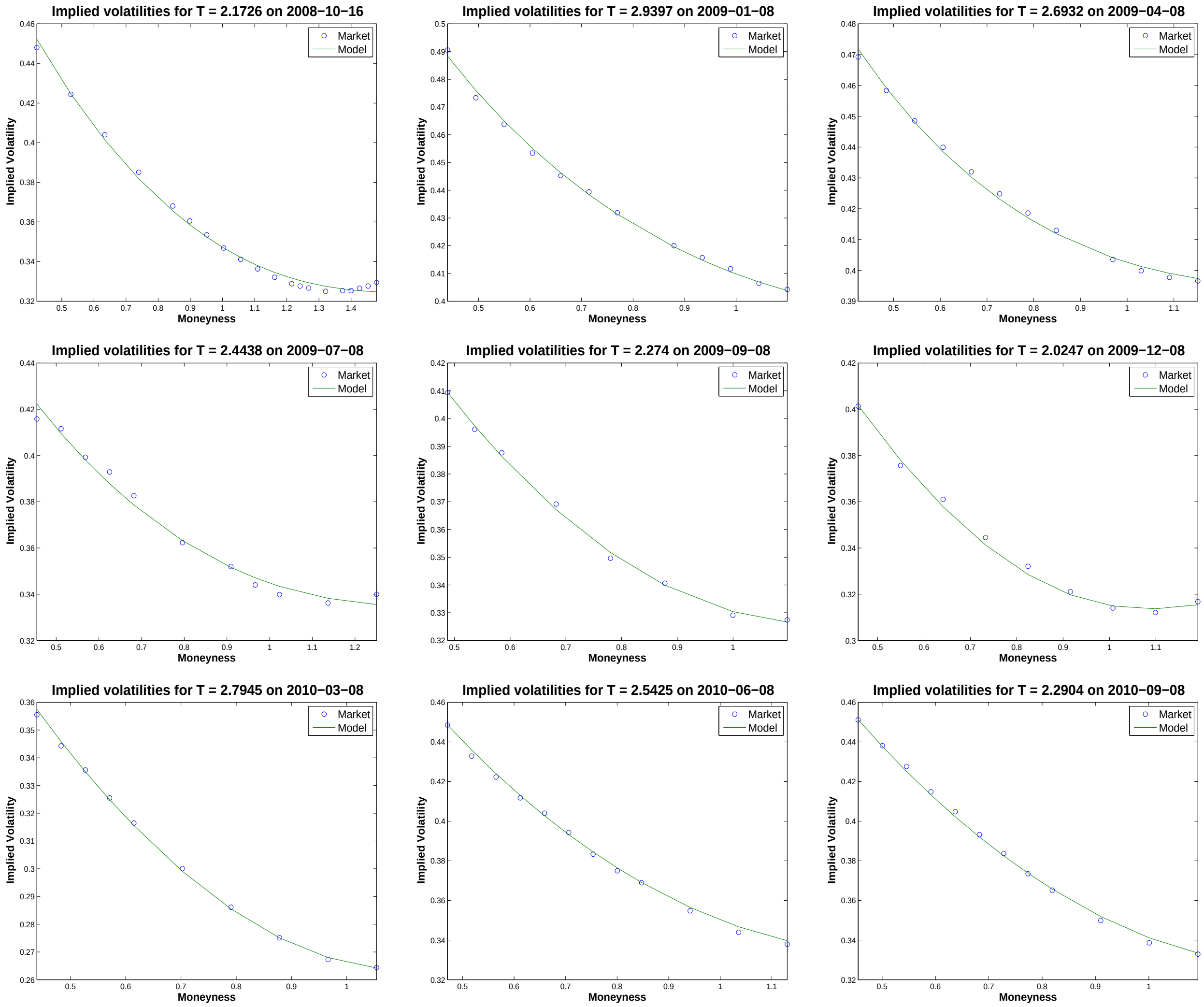

Figure 4: Implied Volatility, Oct. 16, 2008 to Sep. 8, 2010

6.1.2 CDS Spreads

Once we have calibrated the market factor parameters, we then calibrate the idiosyncratic parameters η_i, $i = 1, \ldots, M$, to the (adjusted) market CDS spreads with maturities of 1 and 5 years, following the procedure described in Section 5.1.2. Table 3 displays the average of the 1-year and 5-year spreads from the market and the calibrated model. We generally obtained a good fit for the CDS spreads except for the 1-year spreads on Oct. 31, 2006 and Jul. 17, 2007, but in those cases, the absolute differences were small.

6.1.3 CDO Tranche Spreads

Table 4 compares the 5-year CDO tranche spreads from the market (CDX.NA.IG) and the model for various dates from Aug. 2004 to Sep. 2010. The model spreads are computed from the intensity-based model via the algorithm in Section 3.1, using the calibrated parameters χ_Y and η_i,

	1-yr			5-yr		
	Market	**Model**	**Rel. Diff.**	**Market**	**Model**	**Rel. Diff.**
Aug. 23, 2004	24.62	25.17	2.21%	70.00	69.77	-0.33%
Dec. 5, 2005	14.22	15.53	9.18%	49.00	47.90	-2.25%
Oct. 31, 2006	8.71	11.62	33.45%	34.00	33.01	-2.92%
Jul. 17, 2007	16.93	19.14	13.00%	45.00	44.05	-2.11%
Mar. 14, 2008	138.86	135.98	-2.07%	182.00	185.33	1.83%
Jun. 20, 2008	87.52	88.57	1.20%	115.00	115.38	0.33%
Oct. 16, 2008	155.52	163.64	5.22%	173.00	166.28	-3.89%
Jan. 8, 2009	240.18	240.19	0.01%	220.00	220.02	0.01%
Apr. 8, 2009	295.37	295.54	0.06%	258.00	257.86	-0.05%
Jul. 8, 2009	131.39	131.40	0.01%	139.00	139.00	0.00%
Sep. 8, 2009	135.31	129.53	-4.27%	121.00	128.64	6.31%
Dec. 8, 2009	59.09	61.13	3.46%	98.00	96.90	-1.12%
Mar. 8, 2010	45.75	47.65	4.15%	89.00	88.12	-0.99%
Jun. 8, 2010	111.49	112.06	0.50%	141.00	140.68	-0.23%
Sep. 8, 2010	104.86	103.52	-1.28%	118.00	119.51	1.28%

Table 3: Forward Problem: Average 1-yr and 5-yr CDS Spreads (bps)

$i = 1, \ldots, M$, above. Here, the relative difference is defined as

$$\text{Rel. Diff.} = \frac{Model - Market}{|Market|},$$

where we note that the absolute value in the denominator is used to indicate that the model spread is greater than the market spread when the relative difference is positive whereas the market spread is greater when the relative difference is negative. In terms of the outputs in the table, the tranche spreads that are quoted in percentages (%) are upfront fees while those quoted in bps (no percentage signs) are running spreads; see Section 4.2 for details on the CDX series.

Table 4: 5-year CDO Tranche Spreads

Date (Series)	**Source**	**0-3%**	**3-7%**	**7-10%**	**10-15%**	**15-30%**
Aug. 23, 2004 (Series 2)	Market	40.00%	312.50	122.50	42.50	12.50
	Model	45.84%	394.38	130.05	59.21	13.17
	Rel. Diff.	14.59%	26.20%	6.16%	39.31%	5.37%
Dec. 5, 2005 (Series 5)	Market	41.10%	117.50	32.90	15.80	7.90
	Model	44.53%	128.84	31.37	19.63	7.24
	Rel. Diff.	8.34%	9.65%	-4.66%	24.24%	-8.32%
Oct. 31, 2006 (Series 7)	Market	23.66%	88.26	18.75	7.25	3.43
	Model	23.71%	112.17	14.70	4.60	2.79
	Rel. Diff.	0.19%	27.09%	-21.61%	-36.58%	-18.53%
Jul. 17, 2007 (Series 8)	Market	31.76%	156.36	33.95	17.23	5.59
	Model	36.48%	151.89	13.12	4.73	4.27
	Rel. Diff.	14.88%	-2.86%	-61.35%	-72.55%	-23.62%
Mar. 14, 2008 (Series 9)	Market	67.92%	836.90	462.22	265.56	129.45
	Model	88.60%	2,235.78	735.12	186.26	12.59
	Rel. Diff.	30.44%	167.15%	59.04%	-29.86%	-90.27%

Continued on next page

Table 4 – continued from previous page

Date (Series)	**Source**	**0-3%**	**3-7%**	**7-10%**	**10-15%**	**15-30%**
Jun. 20, 2008 (Series 10)	Market	51.55%	447.32	240.75	124.01	66.66
	Model	76.67%	888.82	165.36	25.12	2.48
	Rel. Diff.	48.73%	98.70%	-31.31%	-79.75%	-96.28%
Oct. 16, 2008 (Series 11)	Market	71.50%	1,297.00	676.67	209.34	66.50
	Model	83.52%	1,637.50	611.51	222.54	25.03
	Rel. Diff.	16.81%	26.25%	-9.63%	6.30%	-62.36%
Jan. 8, 2009 (Series 9)	Market	74.91%	39.75%	811.03	446.13	111.25
	Model	91.90%	51.08%	785.96	255.26	24.86
	Rel. Diff.	22.68%	28.51%	-3.09%	-42.78%	-77.66%
Apr. 8, 2009 (Series 9)	Market	76.76%	45.80%	15.25%	520.21	137.00
	Model	93.47%	58.38%	16.43%	339.31	31.12
	Rel. Diff.	21.76%	27.46%	7.71%	-34.77%	-77.29%
Jul. 8, 2009 (Series 12)	Market	64.00%	34.89%	16.73%	6.80%	-0.83%
	Model	88.61%	44.61%	12.94%	0.84%	-4.05%
	Rel. Diff.	38.45%	27.85%	-22.63%	-87.68%	-387.63%
Sep. 8, 2009 (Series 12)	Market	62.38%	27.31%	10.88%	5.21%	-1.84%
	Model	82.59%	28.28%	5.12%	-1.59%	-4.10%
	Rel. Diff.	32.40%	3.56%	-52.92%	-130.51%	-122.62%
Dec. 8, 2009 (Series 13)	Market	53.42%	22.54%	8.57%	1.75%	-2.44%
	Model	72.47%	26.67%	7.67%	0.21%	-3.85%
	Rel. Diff.	35.67%	18.34%	-10.48%	-87.93%	-57.90%
Mar. 8, 2010 (Series 13)	Market	53.81%	19.75%	7.38%	0.88%	-2.60%
	Model	64.85%	20.74%	5.91%	0.05%	-3.56%
	Rel. Diff.	20.51%	5.01%	-19.97%	-94.69%	-36.83%
Jun. 8, 2010 (Series 9)	Market	52.95%	14.95%	-1.61%	0.81%	-1.48%
	Model	72.36%	8.62%	-9.14%	-1.92%	-2.29%
	Rel. Diff.	36.65%	-42.31%	-467.58%	-336.60%	-54.89%
Sep. 8, 2010 (Series 9)	Market	47.98%	7.23%	-5.78%	-0.52%	-1.71%
	Model	66.26%	2.92%	-9.55%	-1.97%	-2.08%
	Rel. Diff.	38.10%	-59.57%	-65.14%	-279.63%	-21.63%

For further examination, Figures 5 and 6 below show separate time series plots for the equity (0-3%) and senior (15-30%) tranches (after removing the July 8, 2009 outlier). We observe that for the dates between 2004 and 2007, the equity and senior tranche spreads in the market are similar to those in the calibrated model. Hence, this signifies that the systematic risks in equity and credit markets were similar up until 2007. On the other hand, for all of the newer dates since March 2008, the equity tranche spreads observed in the market are much lower than the model spreads, while the senior tranche spreads from the market are significantly larger than the corresponding model spreads. In particular, the credit market accounts for larger losses impacting the senior tranches than as predicted by the model, which is calibrated to equity index options. Hence, we find that in this forward problem, the CDO market contained more systematic risk than was seen in the equity index options market since 2008.

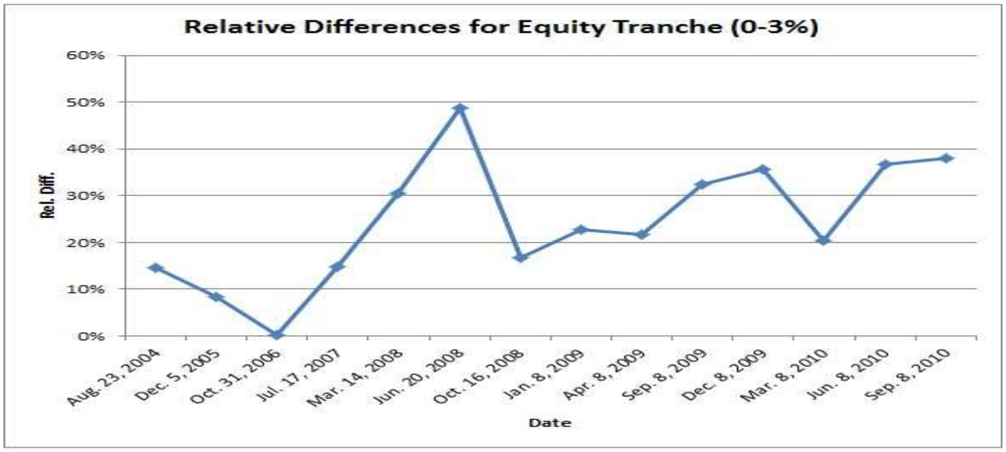

Figure 5: Time Series of Relative Differences for Equity CDO Tranche

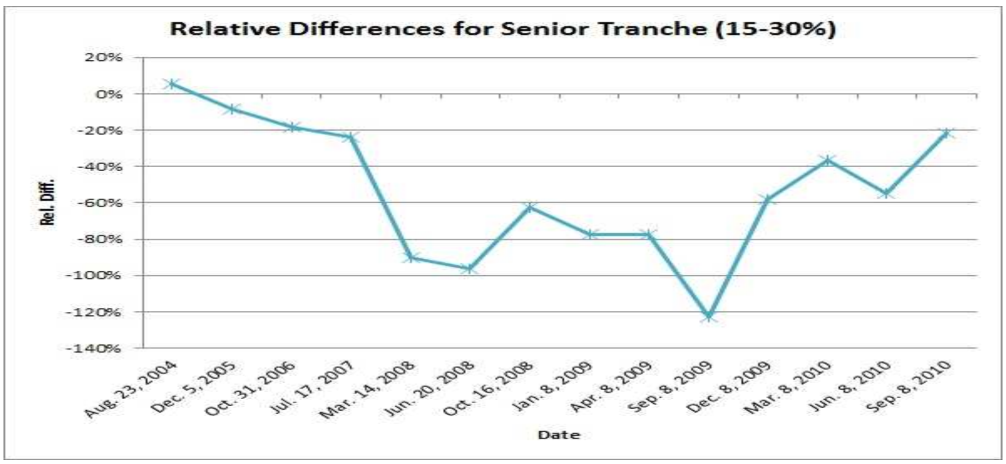

Figure 6: Time Series of Relative Differences for Senior CDO Tranche

6.2 Backward Problem

In this section, we present the numerical results for the backward problem, where we calibrate our model to CDO tranche spreads and CDS spreads and then compare the implied volatilities from equity index options.

6.2.1 Calibrating to CDO and CDS Spreads

Table 5 displays the average 1-year and 5-year CDS spreads from the market and the calibrated model, following the calibration procedure in Section 5.2.1. Note that the market and model CDS spreads are close in most cases, with the exception of the 1-year spreads on Oct. 31, 2006, in which case the absolute difference is small.

In Table 6, we show the market tranche spreads and the model tranche spreads fitted to the data for the period Aug. 2004 to Sep. 2010, following the procedure described in Section 5.2.1. Here, RMSE stands for the root-mean square price error relative to bid/ask CDO tranche spreads

	1-yr			5-yr		
	Market	**Model**	**Rel. Diff.**	**Market**	**Model**	**Rel. Diff.**
Aug. 23, 2004	24.62	25.93	5.31%	70.00	69.38	-0.88%
Dec. 5, 2005	14.22	15.03	5.67%	49.00	48.19	-1.66%
Oct. 31, 2006	8.71	11.26	29.28%	34.00	33.20	-2.37%
Jul. 17, 2007	16.93	17.57	3.77%	45.00	44.47	-1.18%
Mar. 14, 2008	138.86	150.55	8.43%	182.00	167.54	-7.95%
Jun. 20, 2008	87.52	98.47	12.52%	115.00	102.64	-10.75%
Oct. 16, 2008	155.52	167.27	7.55%	173.00	162.14	-6.28%
Jan. 8, 2009	240.18	252.93	5.31%	220.00	204.23	-7.17%
Apr. 8, 2009	295.37	309.91	4.92%	258.00	240.20	-6.90%
Jul. 8, 2009	131.39	139.20	5.94%	139.00	130.06	-6.43%
Sep. 8, 2009	135.31	146.93	8.59%	121.00	108.33	-10.47%
Dec. 8, 2009	59.09	63.22	7.00%	98.00	95.11	-2.95%
Mar. 8, 2010	45.75	49.89	9.06%	89.00	86.65	-2.64%
Jun. 8, 2010	111.49	123.77	11.01%	141.00	127.57	-9.53%
Sep. 8, 2010	104.86	112.04	6.84%	118.00	110.28	-6.54%

Table 5: Backward Problem: Average 1-yr and 5-yr CDS Spreads (bps)

and is computed according to

$$\text{RMSE} = \sqrt{\frac{1}{5}\sum_{k=1}^{5}\left(\frac{R_{k,model} - R_{k,market\,mid}}{R_{k,market\,ask} - R_{k,market\,bid}}\right)^2}, \tag{6.1}$$

where $R_{k,model}$ denotes the model CDO tranche spread for tranche k, $k = 1, \ldots, 5$, and $R_{k,market\,mid}$, $R_{k,market\,bid}$, $R_{k,market\,ask}$ denote the corresponding mid, bid, and ask tranche spreads from the market. Note that the RMSE is generally less than 10 except for one of the dates (i.e., Apr. 8, 2009) during the crisis.

Table 6: Fitting the CDO Tranche Spreads

Date (Series)	**Source**	**0-3%**	**3-7%**	**7-10%**	**10-15%**	**15-30%**	**RMSE**
Aug. 23, 2004 (Series 2)	Market	40.00%	312.50	122.50	42.50	12.50	
	Bid-Ask	2.00%	15.00	7.00	7.00	3.00	
	Model	48.04%	349.62	120.25	60.36	14.96	2.43
Dec. 5, 2005 (Series 5)	Market	41.10%	117.50	32.90	15.80	7.90	
	Bid-Ask	0.80%	6.80	5.30	3.00	1.00	
	Model	42.47%	112.59	30.63	21.48	8.95	1.29
Oct. 31, 2006 (Series 7)	Market	23.66%	88.26	18.75	7.25	3.43	
	Bid-Ask	0.31%	1.25	1.11	0.64	0.97	
	Model	25.72%	88.44	21.54	9.64	3.91	3.60
Jul. 17, 2007 (Series 8)	Market	31.76%	156.36	33.95	17.23	5.59	
	Bid-Ask	0.27%	1.81	1.60	1.61	1.50	
	Model	32.20%	157.53	35.60	17.00	5.79	0.92
Mar. 14, 2008 (Series 9)	Market	67.92%	836.90	462.22	265.56	129.45	
	Bid-Ask	0.56%	9.21	9.07	9.11	4.97	
	Model	67.77%	843.35	440.26	288.30	125.56	1.63

Continued on next page

Table 6 – continued from previous page

Date	Source	0-3%	3-7%	7-10%	10-15%	15-30%	RMSE
Jun. 20, 2008 (Series 10)	Market	51.55%	447.32	240.75	124.01	66.66	
	Bid-Ask	0.78%	7.37	6.50	4.75	3.37	
	Model	51.92%	459.27	226.01	139.51	51.56	2.78
Oct. 16, 2008 (Series 11)	Market	71.50%	1,297.00	676.67	209.34	66.50	
	Bid-Ask	1.50%	50.00	26.67	11.33	10.00	
	Model	77.84%	1,386.39	588.96	269.74	53.74	3.52
Jan. 8, 2009 (Series 9)	Market	74.91%	39.75%	811.03	446.13	111.25	
	Bid-Ask	0.53%	0.41%	8.79	8.79	3.60	
	Model	77.51%	33.94%	830.44	413.34	82.60	7.84
Apr. 8, 2009 (Series 9)	Market	76.76%	45.80%	15.25%	520.21	137.00	
	Bid-Ask	0.34%	0.42%	0.35%	5.45	1.82	
	Model	81.38%	37.37%	12.86%	494.26	132.64	11.51
Jul. 8, 2009 (Series 12)	Market	64.00%	34.89%	16.73%	6.80%	-0.83%	
	Bid-Ask	0.52%	0.53%	0.63%	0.48%	0.25%	
	Model	66.67%	29.97%	15.73%	7.61%	-0.94%	4.86
Sep. 8, 2009 (Series 12)	Market	62.38%	27.31%	10.88%	5.21%	-1.84%	
	Bid-Ask	0.50%	0.50%	0.50%	0.50%	0.29%	
	Model	61.48%	20.77%	7.75%	1.27%	-3.45%	7.83
Dec. 8, 2009 (Series 13)	Market	53.42%	22.54%	8.57%	1.75%	-2.44%	
	Bid-Ask	1.00%	0.78%	0.62%	0.50%	0.40%	
	Model	54.74%	19.45%	9.05%	3.88%	-1.26%	2.99
Mar. 8, 2010 (Series 13)	Market	53.81%	19.75%	7.38%	0.88%	-2.60%	
	Bid-Ask	1.00%	1.00%	1.13%	0.75%	0.50%	
	Model	54.70%	17.92%	7.30%	2.19%	-2.32%	1.22
Jun. 8, 2010 (Series 9)	Market	52.95%	14.95%	-1.61%	0.81%	-1.48%	
	Bid-Ask	0.52%	0.45%	0.43%	0.28%	0.09%	
	Model	52.27%	8.38%	-3.62%	0.96%	-1.98%	7.32
Sep. 8, 2010 (Series 9)	Market	47.98%	7.23%	-5.78%	-0.52%	-1.71%	
	Bid-Ask	0.25%	0.25%	0.25%	0.26%	0.05%	
	Model	47.75%	5.25%	-6.18%	-1.03%	-2.10%	5.10

6.2.2 Market vs. Model Implied Volatilities

Figures 7 to 9 display the implied volatility plots from the market and the model for

$$\rho \in \{-0.30, -0.45, -0.60, -0.75, -0.90\}.$$

For the dates in 2004, 2006, and 2007, we find that the market curve has similar skew to the model curve, and for Dec. 5, 2005, the market curve actually has greater skew. Hence, this signifies that the equity risk was equal to or greater than the credit risk up until 2007. Meanwhile, for all of the newer dates since 2008, we find that the model curves have greater skew than the market curves, and this indicates that the systematic risk in the credit market was greater than that in the equity market for this recent period.

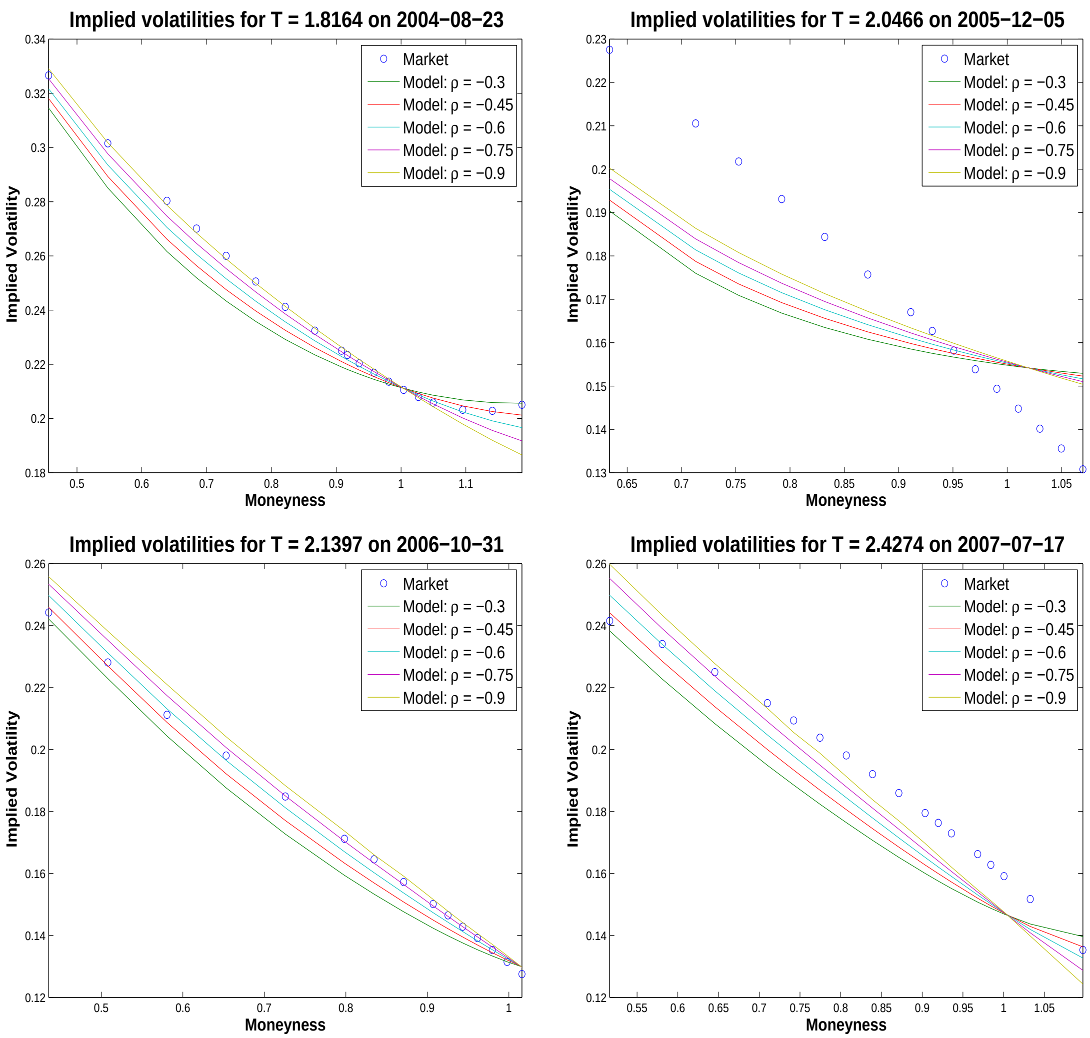

Figure 7: Market vs. Model Implied Vols, Aug. 23, 2004 to Jul. 17, 2007

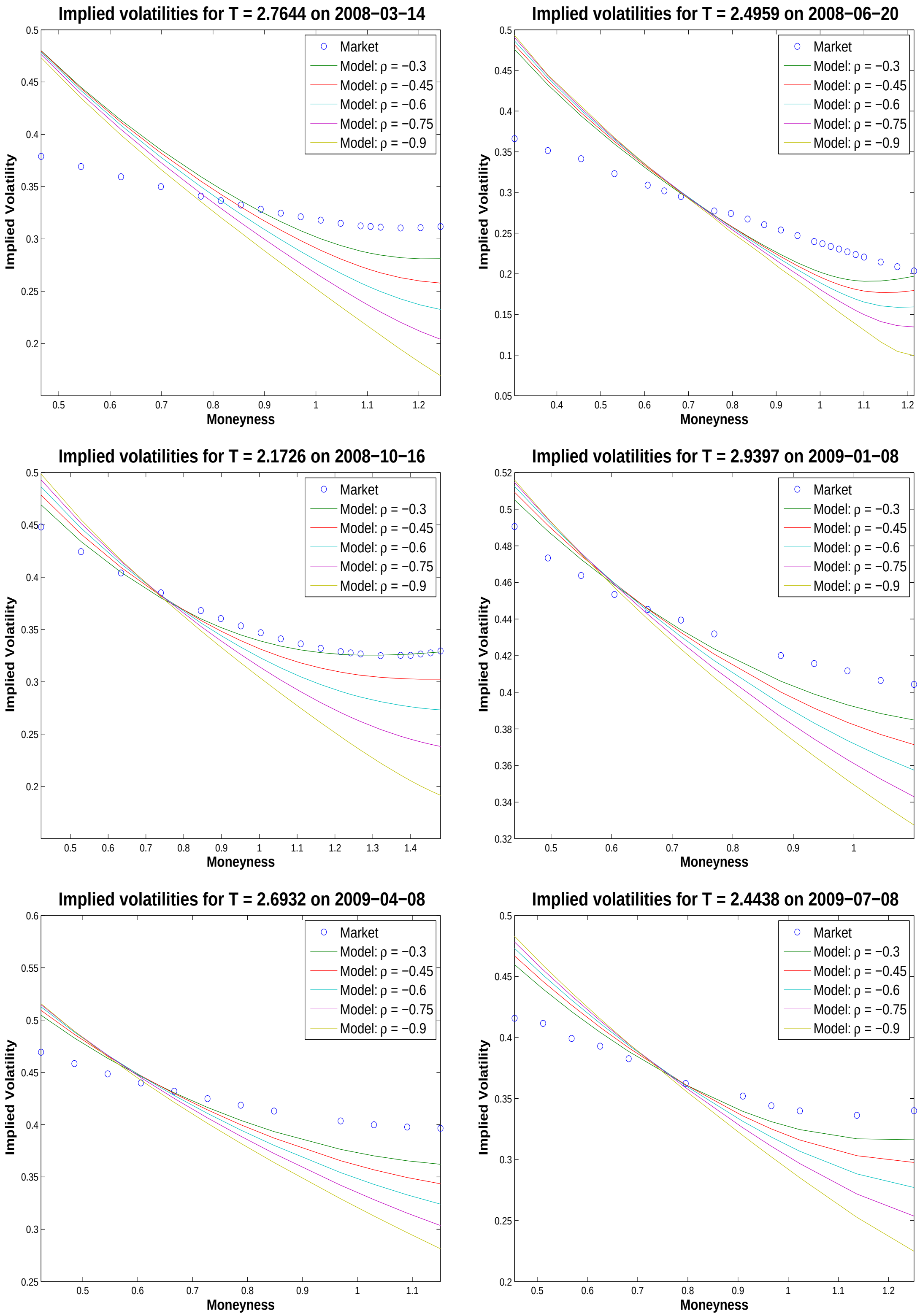

Figure 8: Market vs. Model Implied Vols, Mar. 14, 2008 to Jul. 8, 2009

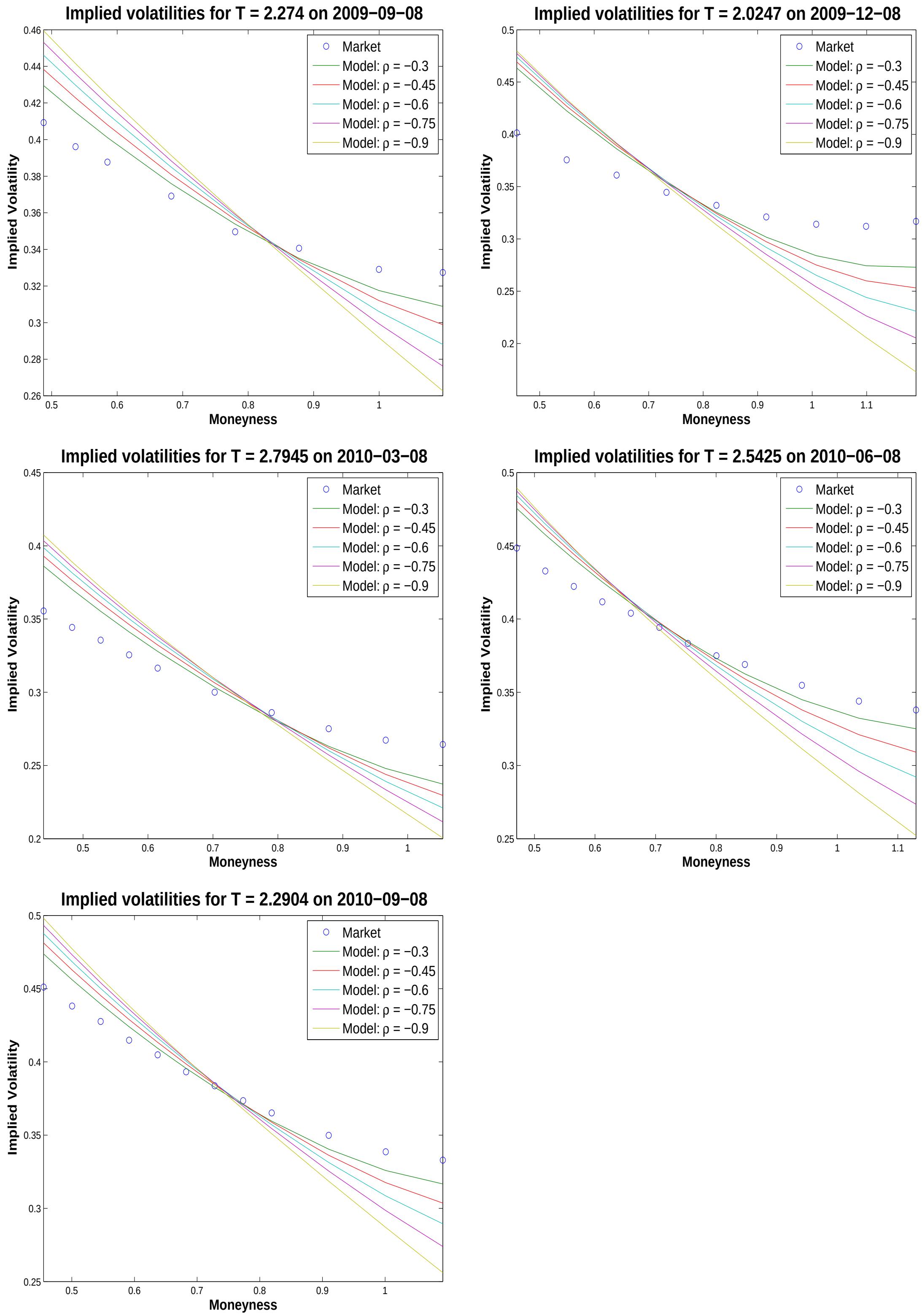

Figure 9: Market vs. Model Implied Vols, Sep. 8, 2009 to Sep. 8, 2010

7 Conclusions

In our hybrid multi-name equity-credit model, we analyzed the systematic risks in equity and credit markets, and we obtained different results before and during the crisis. First, based on pre-crisis data from 2004 to 2007, we found that the systematic risks in equity and credit markets were similar; this result differs from Coval, Jurek, and Stafford [9] but is consistent with Collin-Dufresne, Goldstein, and Yang [8], Luo and Carverhill [21], and Li and Zhao [18]. Meanwhile, we found that during the financial crisis from 2008 to 2010, the credit market incorporated far greater systematic risk than the equity market; we recall that Collin-Dufresne, Goldstein, and Yang [8] and Luo and Carverhill [21] all concluded that the systematic risks were similar based on the crisis data up to Sep. 2008. Let us review our results during the crisis in more detail here.

For the forward problem in Section 6.1, we find that for the dates since 2008, after fitting the model parameters to the equity index options and CDS spreads, the junior CDO tranche spreads from the model are much larger than those observed in the market, while the senior CDO tranche spreads from the model are smaller than the corresponding market spreads. Hence, the credit market accounted for larger losses impacting the senior CDO tranches than as predicted by the model, suggesting that the credit market contained more systematic risk than was seen in the equity market. For the backward problem in Section 6.2, we observed that for the more recent dates since 2008, after fitting the parameters to the CDO and CDS spreads, the implied volatility curves from the model had greater skew than those from the market. This indicates that based on the framework of our model, the systematic risk in the credit market was greater than that in the equity market. Combining the results from the forward and backward cases, we conclude from our hybrid model that the systematic risk in the credit market was greater than that in the equity market during the credit crisis from 2008 to 2010.

Finally, we provide some economic rationale for why the systematic risk in the credit market was larger than that in the equity market during the credit crisis. First, we note that the S&P 500 index option market is more developed than the CDX market and that investors generally have a better understanding of the systematic risks embedded in the equity market, especially during harsh times such as crises. We speculate that investors of credit derivatives would hence be more risk-averse and perhaps pay extra to hedge against the systematic risks in the credit market during the crisis.

A Technical Results for an Integrated Affine Jump Diffusion

In this appendix, we give technical results for an integrated affine jump diffusion (AJD) as it relates to the pricing of CDOs and equity index options for the model in Section 2.1. In particular, we first give the moment generating function and then we provide the characteristic function of an integrated AJD.

We define the integrated AJD by $U_t \equiv \int_0^t Y_s ds$, where $Y = (Y_t)_{t\geq 0}$ is an AJD as defined in Section 2.1. Recall that Y satisfies the SDE

$$dY_t = \kappa(\bar{y} - Y_t)dt + \sigma\sqrt{Y_t}dW_t^Y + dJ_t, \quad Y_0 = y_0, \tag{A.1}$$

where W^Y is a standard $\mathbb{P}^*$-Brownian motion, and J is an independent compound Poisson process with jump intensity l and exponentially distributed jumps with mean ξ. The Laplace transform of the jump size distribution ν is

$$\psi(c) = \int_{\mathbb{R}_+} e^{cz} d\nu(z) = \int_{\mathbb{R}_+} e^{cz} \frac{1}{\xi} e^{-\frac{1}{\xi}z} dz = \frac{1}{1 - c\xi},$$

for $c \in \mathbb{C}$ and $\text{Re}(c) < 1/\xi$.

A.1 Moment Generating Function

By Duffie, Pan, and Singleton [12], it follows that for $t > 0$ and $q \in \mathbb{R}$,

$$\mathbb{E}^* \left[e^{qU_t} \right] = \mathbb{E}^* \left[e^{q \int_0^t Y_s ds} \right] = e^{\alpha(t) + \beta(t) y_0}, \tag{A.2}$$

where $\alpha(\cdot)$ and $\beta(\cdot)$ solve the pair of Riccati ODEs

$$\alpha'(t) = \kappa \bar{y} \beta(t) + l \frac{\xi \beta(t)}{1 - \xi \beta(t)}, \quad \beta'(t) = -\kappa \beta(t) + \frac{1}{2} \sigma^2 \beta(t)^2 + q, \tag{A.3}$$

with boundary conditions $\alpha(0) = \beta(0) = 0$. Appendix B of Duffie and Gârleanu [11] provides an explicit solution, as follows:

$$\begin{aligned} \alpha(t) &= -\frac{2\kappa \bar{y}}{\sigma^2} \log \left(\frac{c_1 + d_1 e^{-\gamma t}}{c_1 + d_1} \right) + \frac{\kappa \bar{y}}{c_1} t + l \left(\frac{d_1/c_1 - d_2/c_2}{-\gamma d_2} \right) \log \left(\frac{c_2 + d_2 e^{-\gamma t}}{c_2 + d_2} \right) + l \frac{1 - c_2}{c_2} t, \\ \beta(t) &= \frac{1 - e^{-\gamma t}}{c_1 + d_1 e^{-\gamma t}}, \end{aligned} \tag{A.4}$$

where

$$\gamma = \sqrt{\kappa^2 - 2\sigma^2 q}, \quad c_1 = \frac{\kappa + \gamma}{2q}, \quad c_2 = 1 - \frac{\xi}{c_1}, \quad d_1 = \frac{-\kappa + \gamma}{2q}, \quad d_2 = \frac{d_1 + \xi}{c_1}. \tag{A.5}$$

Hence, the moment generating function of an integrated AJD is known in closed form.

A.2 Characteristic Function

Setting $q = iv$ for $v \in \mathbb{R}$ in (A.2) gives an explicit formula for the characteristic function of the integrated AJD at time t:

$$\mathbb{E}^* \left[e^{ivU_t} \right] = e^{\alpha(t) + \beta(t) y_0}, \tag{A.6}$$

where $\alpha(\cdot)$ and $\beta(\cdot)$ solve the pair of complex-valued ODEs in (A.3). Repeating the derivation by Duffie and Gârleanu [11] here for the complex-valued case, we arrive at the solution (A.4). In this case, we interpret γ in (A.5) as

$$|\gamma^2|^{1/2} \exp(i \arg(\gamma^2)/2),$$

where for any $z \in \mathbb{C}$, $\arg(z)$ is defined such that $z = |z| \exp(i \arg(z))$ with $-\pi < \arg(z) \leq \pi$. Moreover, we take $\log(z) = \log(|z|) + i \arg(z)$, although any other branch of the complex logarithm would work as well, since the logarithm of γ shows up only in the exponent of (A.6). See Lord and Kahl [20] for a discussion on evaluating transforms of the form (A.6) with a complex-valued exponent.

References

[1] Andersen, L., J. Sidenius, and S. Basu (2003). All your hedges in one basket. *RISK 16(11)*, 67–72.

[2] Bayraktar, E. (2008). Pricing Options on Defaultable Stocks. *Applied Mathematical Finance 15(3)*, 277–304.

[3] Bayraktar, E. and B. Yang (2011). A Unified Framework for Pricing Credit and Equity Derivatives. *Mathematical Finance 21(3)*, 493–517.

[4] Breeden, D. and R. Litzenberger (1978). Prices of State-Contingent Claims Implicit in Option Prices. *Journal of Business 51(4)*, 621–651.

[5] Carr, P. and V. Linetsky (2006). A Jump to Default Extended CEV Model: An Application of Bessel Processes. *Finance and Stochastics 10(3)*, 303–330.

[6] Carr, P. and L. Wu (2010). Stock Options and Credit Default Swaps: A Joint Framework for Valuation and Estimation. *Journal of Financial Econometrics 8(4)*, 409–449.

[7] Collin-Dufresne, P., R. Goldstein, and J. S. Martin (2001). The Determinants of Credit Spread Changes. *The Journal of Finance 56(6)*, 2177–2207.

[8] Collin-Dufresne, P., R. Goldstein, and F. Yang (2012). On the Relative Pricing of Long-Maturity Index Options and Collateralized Debt Obligations. *The Journal of Finance 67(6)*, 1983–2014.

[9] Coval, J., J. Jurek, and E. Stafford (2009). Economic Catastrophe Bonds. *American Economic Review 99(3)*, 628–666.

[10] Duffie, D. (1999). Credit Swap Valuation. *Financial Analysts Journal 55(1)*, 73–87.

[11] Duffie, D. and N. Gârleanu (2001). Risk and Valuation of Collateralized Debt Obligations. *Financial Analysts Journal 57(1)*, 41–59.

[12] Duffie, D., J. Pan, and K. Singleton (2000). Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica 68(6)*, 1343–1376.

[13] Elizalde, A. (2005). Credit risk models IV: Understanding and pricing CDOs. *www.abelelizalde.com*.

[14] Feldhütter, P. (2008). An Empirical Investigation of an Intensity-Based Model for Pricing CDO Tranches. Working paper, Copenhagen Business School.

[15] Feldhütter, P. and M. Nielsen (2010). Systematic and Idiosyncratic Default Risk in Synthetic Credit Markets. Working paper, Copenhagen Business School.

[16] Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *The Review of Financial Studies 6(2)*, 327–343.

[17] Lando, D. (2004). *Credit Risk Modeling: Theory and Applications*. Princeton Series in Finance, Princeton University Press.

[18] Li, H. and F. Zhao (2011). Economic Catastrophe Bonds: Inefficient Market or Inadequate Model? Working Paper.

[19] Longstaff, F. A. and A. Rajan (2008). An Empirical Analysis of the Pricing of Collateralized Debt Obligations. *The Journal of Finance 63(2)*, 529–563.

[20] Lord, R. and C. Kahl (2010). Complex Logarithms in Heston-like Models. *Mathematical Finance 20(4)*, 671–694.

[21] Luo, D. and A. Carverhill (2011). Pricing and Integration of the CDX Tranches in the Financial Market. Working Paper, University of Hong Kong.

[22] Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance 29(2)*, 449–470.

[23] Mortensen, A. (2006). Semi-Analytical Valuation of Basket Credit Derivatives in Intensity-Based Models. *Journal of Derivatives 13(4)*, 8–26.

[24] Papageorgiou, E. and R. Sircar (2009). Multiscale Intensity Models and Name Grouping for Valuation of Multi-name Credit Derivatives. *Applied Mathematical Finance 16(4)*, 353–383.