When Uncertainty and Volatility Are Disconnected: Implications for Asset Pricing and Portfolio Performance[*]

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Abstract

We analyze an environment where the uncertainty in the equity market return and its volatility are both stochastic, generating situations where they can move in tandem as one might expect, but also situations where they are disconnected. We solve in closed form a representative investor's optimal asset allocation and derive the resulting conditional equity premium and risk-free rate in equilibrium. Empirically, we find that equity returns respond negatively to contemporaneous volatility (consistent with the prevalence of the leverage effect in the data), but respond positively, and consistently across horizons, to the uncertainty component in the model. Our results therefore show that the equity premium appears to be earned for facing uncertainty, especially high uncertainty that is disconnected from low volatility, rather than for facing volatility as traditionally assumed. Incorporating the possibility of a disconnect between volatility and uncertainty significantly improves portfolio performance, over and above the performance obtained by conditioning on volatility only.

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"You would think if uncertainty was high, you'd have a bit more volatility."

William Dudley, New York Fed President, February 15, 2017.

1. Introduction

Although the notions of uncertainty and volatility are often used interchangeably, the two concepts are inherently different: volatility measures the dispersion of short-term shocks around a long-term mean, while uncertainty measures the difficulty to forecast more general aspects of the distribution, including its long-term mean. It is natural to expect uncertainty and volatility to be positively correlated. But there have been several episodes in which either volatility was high and uncertainty was low or vice versa. For instance, the US 2016 presidential election generated a high amount of uncertainty about long-term economic and other policies, but surprisingly did not elicit high stock market volatility. Similarly, the UK's exit from the EU (Brexit) involved substantial uncertainty about trade, growth, and immigration policies for the UK and the EU, but had a barely noticeable impact on short-term volatility in their stock markets. These recent events are examples of situations in which a disconnect appeared because uncertainty was substantially higher than volatility. By contrast, the stock market dynamics during the financial crisis in 2008 and the initial stock market reaction to the diffusion of the Covid-19 pandemic in the spring of 2020 illustrate situations in which disconnect occurred due to a higher increase in volatility compared to the rise in associated uncertainty. Interestingly, in the months following the stock market crash in March 2020, volatility declined much faster than uncertainty, leading to a switch in the nature of disconnect, characterized instead by uncertainty being higher than volatility.

Figure 1 contains a scatter plot of volatility and uncertainty at the weekly frequency between January 1986 and December 2020, proxied respectively by realized volatility computed from high-frequency data and the economic policy uncertainty index (EPU) of Baker et al. (2016). The figure shows that the two variables, although generally positively correlated, are far from being perfect substitutes for one another. And the degree of connection between uncertainty and volatility appears to be time-varying, consistent with the anecdotal evidence that can be gleaned from the specific episodes mentioned above.

INSERT FIGURE 1 ABOUT HERE

Motivated by these empirical observations, we construct a novel asset pricing model in which uncertainty and volatility are two separate stochastic processes, which may stochastically switch between being connected and disconnected. In the context of Figure 1, our model delivers a different equity premium and risk-free rate and consequently a different portfolio strategy in regimes in which uncertainty and volatility are high or low at the same time (connected), compared to regimes in which one of them is low while the other is high (disconnected). We will show that incorporating this potential disconnect leads to substantially improved forecasts of the equity premium and portfolio performance.

In existing models that do not account for uncertainty aversion, time-varying volatility drives the optimal consumption and investment policies of a risk-averse agent, but there is no mechanism to account for the degree of uncertainty attached to the assumptions of the model (see, e.g., Chacko and Viceira (2005) and Liu (2007).) On the other hand, in models in which the agent is uncertainty-averse but uncertainty is time-varying while volatility is constant, the optimal strategy is to reduce the exposure to risky assets when uncertainty increases (see, e.g., Maenhout (2004) for an early example of this approach). Since volatility and returns tend to be negatively correlated, the resulting conservative asset allocation makes the investor forgo much of the upside of asset markets in situations in which volatility fails to materialize despite high levels of uncertainty, such as the aftermaths of the US 2016 election, Brexit, and the months that followed the stock market crash of March 2020 driven by the pandemic. Including uncertainty and volatility separately allows the investor to take advantage of such situations.

We model uncertainty using the framework of a robust control problem, following Hansen and Sargent.[1] The novelty in this paper is the addition of a stochastic process that can drive a wedge between volatility and uncertainty, which we label the "disconnect process." We first solve the model in partial equilibrium, computing in closed form the investor's optimal policies for consumption, perturbation function, and portfolio choice. We show that the logarithmic investor's optimal equity holding consists of the standard myopic term, which is inversely proportional to stock return variance, and a new (yet still myopic as befits a logarithmic investor) uncertainty correction term, which is the optimal response to the potential disconnect between volatility and uncertainty. Given the optimal asset allocation by a representative investor, we then solve for the equilibrium equity premium and risk-free rate, which are non-linear functions of both the stochastic volatility and stochastic

[1]The robust control approach of Hansen and Sargent focuses on minimizing the worst case loss over the set of possible models. Recognizing that he is unable to know exactly the true underlying model, the investor seeks instead to develop portfolio policies that should perform reasonably well across the set of alternative models that are statistically close to the reference model in the sense of entropy minimization. Accordingly, the robust investor holds a lower fraction of wealth in the risky asset compared to an investor who is not averse to uncertainty. For more details on the Hansen-Sargent robust control method see Anderson et al. (2003), Hansen et al. (2006), Cogley et al. (2008), Hansen and Sargent (2008), and Hansen and Sargent (2011). For a discussion and comparison of the max-min expected utility of Gilboa and Schmeidler (1989) and robust control theory, see Hansen and Sargent (2001) and Hansen et al. (2006). For asset allocation implications see Trojani and Vanini (2000) and Trojani and Vanini (2004).

disconnect processes. We find that the interaction of volatility and uncertainty plays an important role in determining optimal portfolios. In particular, the sensitivity of portfolio weights to changes in volatility depends on the level of model uncertainty. Accordingly, the trajectory of portfolio weights in a given period depends on the joint dynamics of volatility and uncertainty, including whether they are connected or disconnected.

Most theoretical asset pricing models imply a positive relationship between return and risk.[2] The empirical evidence for such a trade off is, however, mixed or inconclusive, and depends on the sample period and methodology, including whether total or idiosyncratic volatility is considered.[3] Although our model also implies a positive relationship between return and risk, it adds a new component in the equity premium associated to the disconnect between volatility and uncertainty, which is also theoretically positive. Including this component allows our model to more accurately reproduce asset pricing patterns. For example, in high-uncertainty regimes, our model can reproduce high excess returns with simultaneous low stock return volatility.

Overall, we find that once disconnect is possible, stock returns respond negatively to contemporaneous return volatility (consistent with the prevalence of the leverage effect in the data but inconsistent with the predictions of most theoretical asset pricing models), but respond positively, and consistently across horizons, to the uncertainty component in the model. Our results therefore show that allowing for uncertainty to be disconnected from volatility makes uncertainty a much better variable than volatility in terms of generating a trade-off with expected returns: it appears from our results that the equity premium is earned mainly for facing uncertainty, especially high uncertainty that is disconnected from low volatility, rather than for facing volatility per se.

We then evaluate the portfolio performance of a reference investor who predicts future excess returns using our estimated relation between stock excess return, volatility, and uncertainty. We find that our model significantly improves portfolio performance relative to existing unconditional and conditional asset pricing models, including robust ones that allow for the presence of uncertainty but not for its disconnect from volatility, as well as those that time volatility but do not account for its

[2]A growing literature analyzes the effect of uncertainty on the equity premium, both theoretically and empirically. For instance, the theoretical work of Pastor and Veronesi (2012) and Pastor and Veronesi (2013) shows that political uncertainty shocks command a risk premium, and that stocks become more correlated and volatile in periods of elevated political uncertainty. From the empirical side, Brogaard and Detzel (2015) show that uncertainty positively forecasts excess returns and that innovations in uncertainty carry a significant negative risk premium, while Bali et al. (2017) find that the difference between returns on portfolios with the highest and lowest uncertainty beta (seven-factor alpha) is negative and highly significant. The empirical results in both papers are consistent with the notion that investors care not only about expected returns and risk, but also about uncertainty.

[3]See, e.g., Merton (1980), French et al. (1987), Glosten et al. (1993), Goyal and Santa-Clara (2003), Ang et al. (2009) and Campbell et al. (2018).

potential disconnect from uncertainty.[4] These results are valid in and out of sample.

The rest of the paper is organized as follows. Section 2 sets up the model, the investor's problem and its solution in partial equilibrium. Section 3 solves for the equilibrium conditional equity premium and risk-free rate. Section 4 describes the data and our empirical analysis. Section 5 evaluates the performance of our model's implied portfolio strategy in the full sample and in specific high disconnect regimes. Section 6 shows that our results also hold out of sample. Section 7 concludes.

2. Robust portfolio allocation with disconnect

We consider an infinite horizon expected utility maximization problem where the investor chooses his consumption level and allocates his funds between a riskless and a risky asset, which exhibits stochastic volatility. For this purpose, the investor employs a particular model which presumably represents his best estimate of the risky asset's dynamics under a benchmark or reference probability measure. However, the investor fears that the model he uses is potentially misspecified, i.e., he believes that the true model could lie in a larger set of alternative models that are statistically difficult to distinguish from the reference model. To mitigate the effect of potential model misspecification on his utility, the investor chooses optimal consumption and portfolio holdings that are robust against small perturbations of the reference model.

We start by specifying the set of alternative models for asset returns which the investor considers as plausible alternatives. Following the Hansen-Sargent approach, the set of alternative models considered by the investor consists of those models whose relative entropy (or Kullback-Leibler) distance from the reference model is bounded. Models at a small distance from the reference model should be statistically difficult to distinguish from it.

The new element in our approach is that we generalize the upper bound on relative entropy growth. We first let stochastic volatility affect this upper bound rendering the set of alternative models time-varying, i.e., the bound is higher when there is higher volatility and vice versa. To avoid perfect correlation between model uncertainty and volatility we introduce an additional stochastic process that drives uncertainty beyond volatility. This setup allows us to study market scenarios in which, simultaneously, uncertainty can be high while volatility is low and vice versa: those are the "HL" and "LH" regimes characterized in Figure 1. Finally, to solve the optimization problem

[4] Our portfolio strategy is reminiscent of the volatility-managed portfolio allocation in Moreira and Muir (2017), who show that portfolios taking less risk when volatility is high produce large alphas, substantially increase factor Sharpe ratios, and produce large utility gains for mean-variance investors. We show that incorporating disconnect as an additional variable achieves even higher portfolio performance.

we use dynamic programming under constraints to convert the entropy constraint into a penalty on perturbations from the reference model. We then derive the optimal robust solution to the investor's investment and consumption problem in closed form.

2.1 Asset price dynamics under the reference and robust measures

Assume a complete, filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual assumptions. $\mathbb{P}$ denotes the reference (or physical) probability measure. We consider a Lucas tree economy (see Lucas (1978)) with a single risky asset with price S_t and a risk-free asset with price B_t. The risky asset represents a perpetual claim on the stream of aggregate dividends in the economy, whose exogenous dynamics are given by

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D v_t dW_t^D, \tag{1}$$

where μ_D and σ_D are constant parameters. Under the reference measure, the dynamics of the asset prices are

$$\frac{dB_t}{B_t} = r_{f,t} dt, \quad B_0 > 0, \tag{2}$$

$$\frac{dS_t}{S_t} = \left(\mu_{S,t} - \frac{D_t}{S_t}\right) dt + \sigma_S v_t dW_t^S, \quad S_0 > 0, \tag{3}$$

$$dv_t = \kappa_v(\theta_v - v_t)dt + \sigma_v v_t dW_t^v, \quad v_0 > 0, \tag{4}$$

where $r_{f,t}$ denotes the risk-free rate, $\mu_{S,t}$ is the expected total return of the risky asset and v_t is its stochastic volatility. The two Brownian motions are potentially correlated, $\mathbb{E}[dW_t^S dW_t^v] = \rho_{Sv} dt$, where $\rho_{Sv} \in [-1, 1]$ is the correlation coefficient. The stochastic volatility process in Equation (4) follows the SV-GARCH diffusion, where $\kappa_v,\ \theta_v,\ \sigma_v$ are positive parameters denoting the speed of mean reversion, the long run level and the volatility of volatility, respectively.[5]

To introduce the possibility of model misspecification, we specify a set of alternative or worst-case robust dynamics which are statistically close to the reference dynamics in Equation (3). Consider a set of equivalent probability measures $\mathbb{P}^\vartheta$ such that

$$\frac{dS_t}{S_t} = \left(\mu_{S,t} - \frac{D_t}{S_t} + h_t \sigma_S v_t\right) dt + \sigma_S v_t dW_t^{S,\vartheta}, \tag{5}$$

where $W_t^{S,\vartheta} = (W_t^{S,\vartheta})_{t\geq 0}$ is a Brownian motion under $\mathbb{P}^\vartheta$. Equation (5) states that the expected total return on the risky asset has changed from $\mu_{S,t}$ under $\mathbb{P}$ to $\mu_{S,t} + h_t \sigma_S v_t$ under $\mathbb{P}^\vartheta$ where $(h_t)_{t\geq 0}$ is a continuous $\mathcal{F}_t$-measurable function. In what follows, h_t is referred to as the drift perturbation

[5]The choice of the SV-GARCH diffusion process for volatility is essentially without loss of generality for a logarithmic investor.

function. Equation (5) reveals that the effect of model uncertainty in the drift induced by h_t is significantly affected by volatility. That is, low levels of volatility mitigate the impact of uncertainty while high levels of volatility amplify it. Moreover, as we will later show, the optimal drift perturbation h_t will be itself a function of both volatility and uncertainty, thereby inducing a non-linear stochastic stock price drift in equilibrium.

2.2 Introducing disconnect

Equations (3) and (5) imply that the Brownian motions under the reference and robust measures are related by

$$dW_t^S = dW_t^{S,\vartheta} + h_t dt, \tag{6}$$

and from Girsanov's theorem we can determine that the change from the reference to the robust measure is given by the exponential martingale:

$$\vartheta_t = \exp\left\{\int_0^t h_s dW_s^S - \int_0^t \frac{h_s^2}{2} ds\right\}, \tag{7}$$

with dynamics

$$\frac{d\vartheta_t}{\vartheta_t} = h_t dW_t^S. \tag{8}$$

The process ϑ_t restricts the size of alternative models that are statistically difficult to distinguish from the reference model. Given two probability measures $\mathbb{P}$ and $\mathbb{P}^\vartheta$, growth in the entropy of $\mathbb{P}^\vartheta$ relative to $\mathbb{P}$ over the time interval $[t, t+\Delta t]$ is defined as

$$G(t, t+\Delta t) \equiv \mathbb{E}_t^\vartheta\left[\log\left(\frac{\vartheta_{t+\Delta t}}{\vartheta_t}\right)\right], \qquad \forall t \geq 0, \tag{9}$$

where $\mathbb{E}_t^\vartheta[\cdot]$ denotes the conditional expectation up to time t with respect to $\mathbb{P}^\vartheta$. The investor seeks to be robust against departures from $\mathbb{P}$ up to a certain point: the set of admissible model misspecification is restricted to

$$\left\{\vartheta_t : \mathcal{R}(\vartheta_t) \equiv \lim_{\Delta t \to 0} \frac{G(t, t+\Delta t)}{\Delta t} \leq \frac{1}{2}\epsilon^2 \mathcal{U}_t^2,\ \mathcal{U}_t := \eta_t v_t,\ \epsilon \geq 0,\ \eta_t \geq 0,\ v_t \geq 0,\ \forall t \geq 0\right\}, \tag{10}$$

where the stochastic process $\mathcal{U}_t$ represents the amount of uncertainty that the investor is seeking robustness against and the constant parameter ϵ is a measure of the investor's uncertainty aversion: an investor with higher ϵ seeks robustness against a larger set of alternative models. The stochastic process η_t denotes the disconnect process we add to the robust control framework. From Equation (10) we have

$$\eta_t := \frac{\mathcal{U}_t}{v_t} \tag{11}$$

so high levels of η_t capture situations in which the economy is in the "HL" regime, characterized by high uncertainty and low volatility, while low levels of η_t are observed when the economy is in the

"LH" regime, characterized by low uncertainty and high volatility. By contrast, in the two connected regimes "HH" and "LL", η_t tends to be away from extreme values, and closer to its normalized mean value set to 1. Of course, the variables in Equation (11) are continuous so the notion (and number) of discrete regimes is simply a convenient low-dimensional representation of the investment environment that plays no formal role in the analysis of the model.

To fully specify the model, we assume that η_t is a positive process that, like v_t, follows a SV-GARCH diffusion

$$d\eta_t = \kappa_\eta(\theta_\eta - \eta_t)dt + \sigma_\eta \eta_t dW_t^\eta, \tag{12}$$

where $\kappa_\eta,\ \theta_\eta,\ \sigma_\eta > 0$ are positive parameters denoting the speed of mean reversion, the long run level and the volatility of disconnect, respectively.[6] Disconnect and volatility are potentially correlated, with $\mathbb{E}[dW_t^\eta dW_t^v] = \rho_{\eta v}dt$. Accordingly, the stock price and disconnect are also potentially correlated, with $\mathbb{E}[dW_t^S dW_t^\eta] = \rho_{S\eta}dt$.

2.3 Wealth dynamics and optimization problem under robustness

We can now formulate the investor's robust control problem. Let X_t denote the investor's wealth and ω_t be the percentage of wealth (or portfolio weight) invested in the risky asset; $1-\omega_t$ is invested in the risk-free asset. The investor consumes at an instantaneous rate C_t and assumes the risky asset evolves according to the dynamics specified in Equation (5). Accordingly, the dynamics of investor's wealth X_t follow

$$\begin{aligned} dX_t &= \omega_t X_t \left(\frac{dS_t + D_t dt}{S_t}\right) + (1-\omega_t)\, X_t \frac{dB_t}{B_t} - C_t dt \\ &= \left[X_t(r_{f,t} + \omega_t\,(\mu_{S,t} - r_{f,t} + v_t h_t \sigma_S)) - C_t\right] dt + \omega_t X_t v_t \sigma_S dW_t^{S,\vartheta}, \end{aligned} \tag{13}$$

starting from an initial endowment $X_0 > 0$. The investor derives utility $U(C_t)$ from consumption at date t, has subjective discount or impatience rate $\beta > 0$, and solves the infinite horizon problem

$$\max_{\{C_s,\omega_s\}_{t\le s<\infty}} \min_{\{h_s\}_{t\le s<\infty}} \mathbb{E}_t^\vartheta \left[\int_t^\infty e^{-\beta s} U(C_s) ds\right], \tag{14}$$

subject to the wealth dynamics in Equation (13) and the entropy growth constraint in Equation (10). The investor solves this problem taking the dynamics of asset prices as given.

Define the value function

$$V(X_t, v_t, \eta_t) = \max_{\{C_s,\omega_s\}_{t\le s<\infty}} \min_{\{h_s\}_{t\le s<\infty}} \mathbb{E}_t^\vartheta \left[\int_t^\infty e^{-\beta s} U(C_s) ds\right] \tag{15}$$

[6] As it was the case with volatility, the choice of the SV-GARCH diffusion process to model disconnect is essentially without loss of generality in the case of a logarithmic investor, and has no effect on any of our main theoretical or empirical results.

associated with the optimal stochastic robust control problem in Equation (14), where the endogenous drift perturbation function h_t, as we will show below, will be state-dependent in the volatility v_t and disconnect η_t. Since we work in incomplete markets and thus the martingale measure is not unique, we have to rely on standard stochastic dynamic programming. The perturbed Hamilton-Jacobi-Bellman (HJB) equation characterizing the optimal solution is

$$
\begin{aligned}
0 = & \max_{\{C_t,\omega_t\}} \min_{\{h_t\}} U(C_t) + \frac{\partial V}{\partial X}[X_t(r_{f,t} + \omega_t(\mu_{S,t} - r_{f,t} + v_t h_t \sigma_S)) - C_t] \\
& + \frac{1}{2}\frac{\partial^2 V}{\partial X^2} v_t^2 X_t^2 \sigma_S^2 \omega_t^2 + \frac{\partial V}{\partial v}\kappa_v(\theta_v - v_t) + \frac{1}{2}\frac{\partial^2 V}{\partial v^2}\sigma_v^2 v_t^2 + \frac{\partial V}{\partial \eta}\kappa_\eta(\theta_\eta - \eta_t) \\
& + \frac{1}{2}\frac{\partial^2 V}{\partial \eta^2}\sigma_\eta^2 \eta_t^2 + \frac{\partial^2 V}{\partial X \partial v}\omega_t X_t \rho_{Sv}\sigma_S \sigma_v v_t^2 + \frac{\partial^2 V}{\partial X \partial \eta}\omega_t X_t \rho_{S\eta}\sigma_S\sigma_\eta \eta_t v_t + \frac{\partial^2 V}{\partial \eta \partial v}\rho_{\eta v}\sigma_\eta \sigma_v \eta_t v_t,
\end{aligned}
\tag{16}
$$

subject to:[7]

$$
\mathcal{R}(\vartheta_t) = \frac{1}{2}h_t^2 \leq \frac{1}{2}\epsilon^2 \mathcal{U}_t^2,\; \mathcal{U}_t := \eta_t v_t. \tag{17}
$$

The robust optimal control problem is solved in two steps. First, the investor chooses a set of worst-case drift perturbation functions $\{h_s\}_{t \leq s < \infty}$. Second, the investor selects admissible consumption and portfolio holdings $\{C_s, \omega_s\}_{t \leq s < \infty}$ that maximize his expected utility of consumption under the worst-case scenario.

Proposition 1 (Optimal Robust Policies)**.** *With logarithmic utility, the optimal consumption, perturbation function and portfolio policy are given by*

$$
\textit{Consumption}: \quad C_t^* = \beta X_t. \tag{18}
$$

$$
\textit{Perturbation}: \quad h_t^* = -\epsilon \eta_t v_t. \tag{19}
$$

$$
\textit{Portfolio weights}: \quad \omega_t^* = \underbrace{\frac{\mu_{S,t} - r_{f,t}}{\sigma_S^2 v_t^2}}_{\textit{reference demand}} - \underbrace{\frac{\epsilon}{\sigma_S}\eta_t}_{\textit{uncertainty correction}}. \tag{20}
$$

The investor consumes a fraction of wealth proportional to the time-preference parameter β, which is typical of logarithmic utility. Interestingly, the optimal drift perturbation function h_t^* is driven by the interaction of stochastic disconnect and volatility, and is further amplified by uncertainty aversion.[8] This implies that, for a given level of uncertainty aversion, the drift adjustment can be high even when volatility is low, if there is high disconnect (HL regime). The second term in the demand function for the risky asset in Equation (20) represents the adjustment made by the investor to his portfolio holdings in the presence of disconnect. It is negative and increasing in magnitude in the investor's degree of uncertainty aversion and in the level of disconnect. The correction term is

[7] Appendix A.1 shows that $\mathcal{R}(\vartheta_t) = \frac{1}{2}h_t^2$.

[8] While in principle there are two roots for the solution of $h_t^* = \pm\epsilon\eta_t v_t$, we focus on the realistic solution with $h_t^* = -\epsilon\eta_t v_t < 0$, which prevails when the expected return of the risky asset under the reference measure is positive.

largest when η_t is largest, i.e., in the HL regime where uncertainty is high and volatility is low, and lowest when η_t is lowest, i.e., in the LH regime characterized by low uncertainty and high volatility. In general equilibrium, the risk premium will adjust to induce the investor to nevertheless hold the risky asset. For this to happen, the risk premium must be relatively high in the HL regime and lower in the LH regime, which turn out to be key predictions of the model that match the empirical evidence.

3. General equilibrium

An equilibrium is a specification of the dynamics of the risky and riskless asset prices, i.e., $\mu_{S,t} = \mu_S(v_t, \eta_t)$ and $r_{f,t} = r_f(v_t, \eta_t)$ combined with a set of optimal (robust) consumption and investment policies that support continuous clearing in the markets for the consumption good and the risky asset:

$$C_t = D_t, \tag{21}$$

$$\omega_t = 1. \tag{22}$$

The next proposition characterizes the equilibrium risk-free rate $r_{f,t}$ and the conditional equity premium under the reference measure, which can be expressed as

$$\frac{1}{dt}\mathbb{E}_t\left[\frac{dS_t + D_t dt}{S_t}\right] - r_{f,t} = \mu_{S,t} - r_{ft}, \tag{23}$$

given the dynamics for the stock price S_t in Equation (3).

Proposition 2. *In equilibrium, the price-dividend ratio is* $\frac{S_t}{D_t} = \frac{1}{\beta}$ *and under the reference measure* $\mathbb{P}$*, the equilibrium equity premium and risk-free rate are given by*

$$\textit{Equity premium}: \quad \mu_{S,t} - r_{f,t} = \sigma_D^2 v_t^2 + \epsilon\sigma_D \eta_t v_t^2. \tag{24}$$

$$\textit{Risk-free rate}: \quad r_{f,t} = \beta + \mu_D - \sigma_D^2 v_t^2 - \epsilon\sigma_D \eta_t v_t^2. \tag{25}$$

From Proposition 2 we observe that the equity premium and risk-free rate are time-varying and non-linear functions of volatility and disconnect. In the benchmark case of no uncertainty aversion, i.e. $\epsilon = 0$, stochastic volatility generates a flight-to-quality-like correlation among asset returns. In turbulent times the demand for risky assets decreases and the demand for safe assets increases (decreasing their yields). Adding uncertainty further amplifies these effects in a non-linear fashion. In particular, Equation (24) implies that, in a low volatility environment, the equity premium may still be large provided that uncertainty is high, i.e. η_t is high. Similarly, Equation (25) implies that the risk-free rate goes down when disconnect is high, even in a low volatility environment.

The equilibrium equity premium and risk-free rate can be also understood in the context of the optimal portfolio policy obtained in Proposition 1. The negative uncertainty correction implies that the representative investor is more conservative and prefers to hold less of the risky asset and more of the risk-free asset compared to a reference investor. In general equilibrium, he must allocate all his wealth to the risky asset. Therefore, the last term in the equity premium boosts up the reward from holding the risky asset just enough so that he optimally chooses to allocate all his wealth to the risky asset. By the same token, the risk-free rate has to decrease in general equilibrium, so the investor optimally chooses not to hold the risk-free asset at all.

4. Empirical analysis

4.1 Data

The weekly S&P 500 (logarithmic) returns including dividends obtained from the CRSP database serve as a proxy for the risky asset return. The data period is from January 1, 1986 until December 31, 2020. We estimate the diffusive term of the stock price process $\sigma_{S,t} := \sigma_S v_t$, with $\sigma_S = 1$ since we match v_t to the square root of realized variance, which is computed daily from aggregated intraday returns sampled at five minutes frequency (78 observations per trading day). To obtain a weekly measure of realized variance, we sum the daily realized variance estimates. The risk-free rate is the three-month (constant maturity) Treasury bill rate which we obtain from the Fred St. Louis Database (Ticker "DGS3MO").[9]

We proxy uncertainty using the economic policy uncertainty (EPU_t) index developed by Baker et al. (2016).[10] The EPU is based on textual analysis of reports by the Congressional Budget Office (CBO) that compile lists of temporary federal tax code provisions and forecast dispersion about future inflation.[11] Based on the definition of disconnect in Equation (11), we construct an empirical proxy for η_t by computing the ratio of uncertainty as measured by EPU to realized volatility. The resulting disconnect process is then normalized to have mean one. Recall that when η_t is far away

[9] In our sample, there are 77 dates for which there is no three-month Treasury bill data available. These dates roughly correspond to the Martin Luther King Jr. Day, President's Day and Columbus Day. For those dates, we proxy the three-month Treasury bill rate by its quote on the previous day.

[10] The daily EPU time series is substantially more volatile than its weekly counterpart. We smooth the daily EPU series by computing its moving average over the past week (5 trading days) to construct our proxy for uncertainty at the weekly frequency.

[11] There are other possible measures of uncertainty. For example, the uncertainty index of Jurado et al. (2015) is more correlated with realized volatility than the EPU is, and would therefore be less appropriate for capturing possible disconnect. It is worth noting that our disconnect measure is positively correlated (0.56) with the ambiguity index developed in Brenner and Izhakian (2018). We thank Yud Izhakian for sharing his data with us.

from one there is high disconnect between uncertainty and volatility, and when η_t is close to one there is low disconnect.

INSERT TABLE 1 HERE

Table 1 summarizes key asset pricing moments for the full sample and for three subperiods around which we will examine the performance of the model: Financial crisis (from July 2017 until March 2009), US 2016 election (from July 2016 until January 2018), and Covid-19 (from January 2020 until December 2020). In the full sample, the equity premium is 7.75%, despite a relatively high level of the risk-free rate. The financial crisis period, by contrast, is associated with a significantly negative return (-35.42%) and higher volatility. During the crisis, disconnect was about half its level in the full sample, as volatility increased much more than uncertainty. The period surrounding the US 2016 election was characterized by a higher excess return (about double its level in the full sample) and much lower volatility (about half its level in the full sample). Because uncertainty was elevated while volatility remained low, disconnect was high in this period (about 1.5 times its level in the full sample). Finally, the Covid-19 period was characterized by high excess returns, similar to those around the US 2016 election, but with considerably high volatility, comparable to the one experienced in the financial crisis. Despite this high level of volatility, disconnect was very high (about 1.5 times its level in the full sample), in contrast to its very low level during the financial crisis. This implies that uncertainty was lower than volatility during the financial crisis, but higher than volatility during Covid-19. Interestingly, while disconnect was on average high around Covid-19, it fluctuated significantly, with low disconnect in March 2020 when financial markets were hit most, and with high disconnect in the following months, when financial markets recovered.

INSERT FIGURE 2 ABOUT HERE

In Figure 2, we plot the time series of our volatility and uncertainty proxies, as well as the resulting disconnect processes. From Panel A, we observe that uncertainty is substantially more volatile than volatility: the standard deviation of the EPU index is 50.1% compared to only 7.3% for realized volatility. While less volatile, realized volatility is more persistent than uncertainty: the first order autocorrelation of volatility is 0.87, compared to 0.74 for the EPU index. In Panel B, we plot the extracted normalized disconnect time series η_t. Since it is constructed as a normalized ratio of uncertainty and volatility, it is also volatile and persistent. High disconnect occurs whenever η_t is far from 1: $\eta_t > 1$ corresponds to a regime of high uncertainty and low volatility, while $\eta_t < 1$ corresponds to a regime of low uncertainty and high volatility. By contrast, when η_t is close to 1, disconnect is low: uncertainty and volatility are either both low, or both high. The early 1990s, the period surrounding the US 2016 presidential election, and most of 2020 (characterized by the pandemic) stand out as episodes of high uncertainty and low volatility. By contrast, the dotcom bubble burst in 1999-2000 and the financial crisis in 2008 are episodes when volatility increased more than uncertainty.

Consistent with Figure 1, the correlation between volatility and uncertainty is only 0.44. This reinforces the idea that, while the two series comove on average, they are sometimes disconnected. For instance, regressing uncertainty on volatility and a constant we obtain an adjusted R^2 of about 20%. This implies that about 80% of the time series variation in uncertainty cannot be explained by volatility – the gap which we attribute to disconnect in the model.

4.2 Equity premium and risk-free rate in different regimes

Proposition 2 implies that the equity premium and the risk-free rate are functions of v_t and η_t. While these variables are continuous in the model, it is useful for interpretation purposes to first think in terms of discrete regimes corresponding to ranges of values: High volatility with high uncertainty (HH), high volatility with low uncertainty (HL), low volatility with high uncertainty (LH), and low volatility with low uncertainty (LL), as described in Figure 1. We define threshold levels for v_t and η_t, given by their mean plus one half of their standard deviation.[12] We call observations above the threshold level "high," and observations below the threshold level "low."

Table 2 reports the average stock market excess return and risk-free rate at the two-week horizon in each regime. It also shows the estimated unconditional probability of being in each regime.

INSERT TABLE 2 ABOUT HERE

The most frequent regime is LL (both volatility and uncertainty are low, η close to one), characterized by a high risk-free rate and a low average stock excess return. Regime HH (both volatility and uncertainty are high, η close to one) exhibits the lowest level of the risk-free rate, and considerably higher stock excess return than in the LL regime. Moving on to the disconnected regimes, we find that the average stock excess return is highest in the HL regime (high uncertainty with low volatility, high η) while the average stock excess return on the LH regime (low uncertainty with high volatility, low η) dominates only those in the LL regime. This is precisely what the model implies: the stock market provides a sizable compensation for bearing uncertainty, especially in regimes in which uncertainty is high and disconnected from volatility (HL regime, high η), compared to the premium earned for bearing volatility. The results in Table 2 correspond to a fixed investment horizon of two

[12]The analysis presented here is robust to alternative thresholds definitions, which we omit for brevity.

weeks; we now turn to studying the realized stock excess returns for investment horizons ranging from $\tau = 1$ to 12 weeks. Figure 3 shows the results.

INSERT FIGURE 3 ABOUT HERE

Panel A in Figure 3 plots the stock excess return for each regime as a function of the investment horizon τ. Consistent with our results in Table 2, high-uncertainty regimes (HH and HL) deliver high subsequent stock excess returns for all the horizons considered. The stock excess return in low-uncertainty regimes (LL and LH) are also positive for most horizons (with the only exception for $\tau = 12$ in the LH regime), but they are significantly lower in magnitude compared to those achieved in high-uncertainty regimes (HH and HL). To incorporate the volatility dimension in the comparison, Panel B plots the Sharpe ratios, obtained by scaling the average realized stock excess returns by their respective standard deviation. By this metric, the HL regime achieves the highest performance for most horizons (with the only exception for $\tau = 11$ weeks), outperforming the results achieved in regime HH. It then follows that, while high-uncertainty regimes offer a high subsequent stock excess return (Panel A), regimes with high uncertainty and simultaneous high volatility deliver subsequent more volatile excess returns, leading to investments in regime HL outperforming those in regime HH for any risk-averse investor. This is a consequence of volatility persistence (as discussed above, volatility is highly persistent, and in particular more persistent than uncertainty).[13]

Next, we examine the predictions of the model by regressing the conditional equity premium given in Proposition 2 on both the variance (v_t^2) and uncertainty term ($v_t^2 \eta_t$), using the continuum of their values and without introducing discrete regimes. Figure 4 shows the results.

INSERT FIGURE 4 ABOUT HERE

Panel A in Figure 4 reports the estimated coefficient from regressing the stock excess return on realized variance (v_t^2) only, with the shaded area representing a 95% confidence interval. It shows that there is a positive albeit insignificant relationship between the stock excess return and its volatility. In Panel B we do a similar analysis, but adding the uncertainty term ($v_t^2 \eta_t$) to the regression, as specified by our result in Proposition 2. We obtain that both volatility and the uncertainty term are statistically significant with 5% significance level (except for short horizons up to two weeks). The estimated regression coefficient for the variance term is negative, in line with the leverage effect. For the uncertainty term, however, the sensitivity is positive and consistent with the model's prediction. Finally, the bottom panel shows that our model including both volatility and uncertainty delivers

[13]We replicated Figure 3 for the risk-free rate and found that it is generally insensitive to changes in the horizon.

a substantially higher (in-sample) R^2. Comparing panels A and B, it appears that volatility as the sole state variable struggles to generate the variation in the equity premium that is present in the data; the effect of volatility on excess returns in the data is not as clear cut as standard financial theory suggests. By averaging periods when the trade-off between volatility and return works in the expected direction with periods when it does not, the regression in Panel A ends up with a still positive, but barely significant effect. Once we include both volatility and uncertainty in Panel B, volatility is no longer tasked with playing this dual role and we obtain a clean separation of the effect of volatility (negative) and uncertainty (positive) on the equity premium. Summing up, we find fairly strong support in the data for the model's predictions as given in Proposition 2, with the provision that volatility has a negative effect on the equity premium.

5. Disconnect-managed portfolios

From Proposition 2, the model-implied equity premium is given by:[14]

$$\mathrm{ERP}_t = \mu_{S,t} - r_{f,t} = v_t^2 + \epsilon \eta_t v_t^2. \tag{26}$$

This implies that a forecast for the equity premium can be obtained from the regression

$$r_{e,t+\tau} := r_{t,t+\tau} - r_{f,t,t+\tau} = \beta_0 + \beta_v v_t^2 + \beta_\eta \eta_t v_t^2 + \epsilon_{t+\tau}, \tag{27}$$

for any horizon τ. Estimating this equation via OLS we obtain

$$\widehat{r}_{e,t+\tau} = \widehat{\beta}_0 + \widehat{\beta}_v v_t^2 + \widehat{\beta}_\eta \eta_t v_t^2 \tag{28}$$

where $\widehat{r}_{e,t+\tau}$ represents the estimated equity premium at horizon τ. Finally, we plug this expression into the optimal policy of a standard logarithmic reference investor, which gives us for that investor an optimal estimated portfolio weight of the form

$$\widehat{\omega}_t^* = \frac{\mu_{S,t} - r_{f,t}}{v_t^2} = \frac{\widehat{\beta}_0}{v_t^2} + \widehat{\beta}_v + \widehat{\beta}_\eta \eta_t \tag{29}$$

Accordingly, in this section we try to assess whether a reference logarithmic investor could achieve better portfolio performance by taking into account that the equity premium is a function of both volatility and uncertainty, as suggested by our model in Proposition 2.

For a standard model with stochastic volatility but no uncertainty, we re-estimate Equation (27) but regressing on the volatility term only. For the portfolio performance analysis, we multiply

[14]Since we estimate the diffusive part of the stock price process via the realized volatility estimator, we set $\sigma_S = \sigma_D = 1$.

the estimated portfolio weights in Equation (29) by a constant $L = 0.15$, which implies that both dynamic portfolio strategies are on average about 100% invested in the index.

We now turn to comparing portfolio strategies over the full sample, and over sub-periods representing different volatility and uncertainty regimes. We compare an unconditional, passive, buy-and-hold indexing strategy, $\omega_t^I = 1$, a portfolio strategy in which the stock excess return is obtained by conditioning on volatility only, and a portfolio strategy in which the stock excess return is obtained conditioning on both volatility and uncertainty. We highlight that all the portfolio comparisons in this section correspond to a full in-sample analysis with the advantage of hindsight. In other words, we back-test all the portfolio strategies considered. In section 6 below, we show that adding uncertainty as a predictor of future stock excess return improves forecast accuracy out-of-sample, over and above the forecast accuracy that can be obtained by using only volatility as a predictor.

5.1 Full Sample

We start the portfolio analysis by comparing the above mentioned three different asset allocation strategies (unconditional, conditional on volatility only, conditional on volatility and uncertainty) over the full sample January 1986 - December 2020, at horizons of two and four weeks.[15] Panels A and B in Figure 5 plot the time series of optimal portfolio weights, while panels C and D report the resulting cumulative wealth processes.

INSERT FIGURE 5 ABOUT HERE

The dotted black line represents the unconditional passive buy-and-hold indexing strategy, the dashed blue line represents the active portfolio strategy conditioning on volatility only, and the solid red line represents the active portfolio strategy conditioning on both volatility and uncertainty. Panels A and B show that both active strategies use leverage extensively, as the portfolio holdings ω_t are very often above one. The two active portfolio strategies are correlated, consistent with the idea that on average volatility and uncertainty are connected and hence comove. They diverge mainly in periods of high disconnect. For example, in the period around the financial crisis characterized by high volatility and simultaneous high disconnect implied by low uncertainty (LH regime), the active strategy conditioning on both volatility and uncertainty was more conservative than the one conditioning on volatility only, thereby avoiding the significant stock market losses in this period. By contrast, in the period around the US 2016 election which was characterized by high uncertainty and simultaneously low volatility (HL regime), hence high disconnect, the active strategy conditioning

[15]Different horizons of up to 12 weeks deliver similar results.

on both volatility and uncertainty was more aggressive than the one conditioning on volatility only, which resulted in taking advantage of the significant stock market gains during this period. Panels C and D show that the portfolio strategy based on the model (conditioning on both volatility and uncertainty) achieves the highest associated wealth compared to both a strategy conditioning on volatility only and an unconditional passive indexing strategy.

We then compute the wealth excess return associated with each portfolio strategy and evaluate their risk and performance based on the following statistics: (i) average wealth excess return, (ii) standard deviation of the wealth excess return, (iii) maximum draw-down $\left(\mathcal{MDD}^{\%}\right)$,[16] (iv) Value-at-Risk (VaR) at confidence level 5%, (v) Sharpe ratio, and (vi) Treynor ratio. Figure 6 summarizes key return statistics.

INSERT FIGURE 6 ABOUT HERE

Panel A of Figure 6 shows that the active portfolio strategy conditioning on both volatility and uncertainty achieves higher excess return on average than the portfolio strategy conditioning on volatility only and the unconditional passive investment strategy for all horizons. Panel B shows that this is not due to excessive risk taking as the standard deviation of the three strategies considered are close to one another, and none of them exhibits the lowest standard deviation across all horizons. Panels C and D show that the portfolio strategy conditioning on both volatility and uncertainty preserves the investor's wealth very effectively, as it achieves the lowest maximum drawdown over most horizons and the lowest value-at-risk in all horizons considered. Finally, panels E and F show that the portfolio strategy conditioning on both volatility and uncertainty delivers the highest risk-adjusted return, as it achieves the highest Sharpe and Treynor ratios for all horizons considered.

We now evaluate the idiosyncratic risk (α) and the systematic risk (β) components for each portfolio strategy. Figure 7 summarizes the results.

INSERT FIGURE 7 ABOUT HERE

Panel A in Figure 7 shows that the active portfolio strategy conditioning on both volatility and uncertainty achieves the highest (CAPM) α for all horizons considered. Panel B shows that

[16] Let W_t denote the current level of wealth process and let $\overline{W}_t := \max_{s\in[0,T]} W_s$ be its running maximum. The drawdown at time t measures the drop of wealth from its running maximum, and is defined as $\mathcal{D}_t := \overline{W}_t - W_t$, $\forall t \geq 0$. The maximum draw-down at time t denoted by $\mathcal{MDD}_t$ records the worst performance that could have happened in the past. It is defined as the running maximum of the drawdown process, given by $\mathcal{MDD}_t := \max_{\tau\in(0,t)} \mathcal{D}_\tau$. Finally, to express the maximum drawdown as the highest proportional wealth loss in the past, we compute $\mathcal{MDD}^{\%} = \mathcal{MDD}_t/\overline{W}_t \in (0,1)$.

the overall systematic risk β is similar for both active strategies, with none of them achieving a lower β for all horizons considered. Interestingly, all the estimated β coefficients are lower than one, which indicates that the large reported α are not driven by excessive systematic risk taking.[17] Finally, panels C and D show that the active portfolio strategy conditioning on both volatility and uncertainty achieves the highest α for all horizons considered, whether we use CAPM (Panel A), Fama-French three-factor (Panel C), or five-factor (Panel D) models as benchmark.

5.2 High Disconnect Regimes

Figure 2 clearly established that there are several prominent disconnect periods in recent financial markets history. In addition, Table 2 suggests that the impact of disconnect on stock excess return can be very different depending on whether it is driven by low uncertainty and high volatility (LH) or by high uncertainty and low volatility (HL). In particular, subsequent stock excess return tends to be higher in HL regimes than in LH regimes, as predicted by the theoretical model.

In this subsection, we analyze portfolio performance from following the same three strategies in specific periods of high disconnect, including both LH regimes and HL regimes.

5.2.1 LH regimes

We start by analyzing a single episode that occurred in a high-disconnect regime, characterized by low uncertainty and high volatility: the 2008 financial crisis.

Financial Crisis

Table 1 shows that over the two-year time period from July 2007 until March 2009 the average stock market excess return was -35.42% and the stock market volatility was 25.48%. Accordingly, the excess return of wealth associated with all portfolio strategies considered were negative around this period. To facilitate the analysis, in this subsection we replace Sharpe and Treynor ratios (which are not straightforward to interpret for ranking purposes when average excess returns are negative) by the average and standard deviation of wealth excess returns.

INSERT FIGURE 8 ABOUT HERE

[17] In results not reported here, we find that the R^2 obtained from a regression of the wealth excess return for each active portfolio strategy onto the stock market return are 0.6 (conditioning on volatility only) and 0.5 (conditioning on volatility and uncertainty), respectively.

Panels A and B on Figure 8 show that the active portfolio strategies achieve a higher terminal wealth than the index during this sub-period. At the two-weeks horizon, the wealth evolution associated to conditioning on volatility only is very similar to the one associated to conditioning on both volatility and uncertainty (Panel A). However, when considering a one-month horizon, the portfolio conditioning on volatility and uncertainty generates the highest associated level of wealth (Panel B).

Panel C shows that the active portfolio strategies achieve higher excess return on average than the passive index strategy for all horizons. However, there is no clear dominant active strategy based on this metric, as none of them achieves the highest wealth excess return consistently across all horizons considered. Panel D shows that the higher excess return achieved by the active strategies is not due to excessive risk taking, as the standard deviation of their wealth excess return is lower than the one associated with the passive index strategy. The standard deviation of wealth excess return associated with the active strategies are very similar to one another for all horizons. Panels E and F show that the active portfolio strategies do equally well in wealth preservation, as they obtain similar levels of maximum drawdown and value-at-risk in most horizons. By contrast, the unconditional passive index strategy is less effective at wealth preservation, as its levels of maximum drawdown and value-at-risk are about double their corresponding levels for its active portfolio strategy counterparts at all horizons.

5.2.2 HL regimes

Below we analyze two episodes that occurred in high-disconnect regimes, characterized by high uncertainty and low volatility: the US 2016 election, and the Covid-19 crisis.

US 2016 election

From roughly mid of 2016 until early 2018, uncertainty remained at elevated levels while realized volatility reached record lows. In Figure 9 we plot the wealth evolution along with risk-adjusted returns for each portfolio strategy around the US 2016 election. Specifically, here we consider the period from July 2016 until January 2018.

INSERT FIGURE 9 ABOUT HERE

Panels A and B show that the conditional active portfolio strategies achieve a higher terminal wealth than the unconditional indexing strategy during this sub-period. At the two-weeks horizon the portfolio conditioning on volatility and uncertainty generates the highest associated level of wealth

(Panel A). When considering a one-month horizon, the wealth evolution associated to conditioning on volatility only is similar to the one associated to conditioning on both volatility and uncertainty. Panels C and D show that the portfolio conditioning on both volatility and uncertainty achieves the highest risk-adjusted returns in most situations (with the only exception of the one-month horizon, in which conditioning only on volatility slightly dominates). In addition, it is worth noting that while both active strategies outperform the index, this is partly due to the use of high leverage (the portfolio weights for the active strategies tend to be high, sometimes even above 2). Finally, panels E and F show that the active portfolio strategies would not preserve the investor's wealth effectively over this period, as they obtain higher maximum drawdowns and value-at-risk than the passive index across all horizons. Comparing the two conditional active strategies only, conditioning on both volatility and uncertainty does a better job at wealth preservation, achieving lower maximum drawdown and value-at-risk across all horizons.

Covid-19

The Covid-19 pandemic triggered strong market reactions. At the peak of the crisis in mid March 2020, stock market volatility even exceeded the highest levels previously seen during the financial crisis period and caused markets to crash within a few weeks. This period is particularly interesting to study within our framework, because of the different timing in the dynamics of volatility and uncertainty. Uncertainty increased substantially in late February and remained at elevated levels throughout most of the sample period. By contrast, volatility initially spiked sharply and then quickly mean reverted. In terms of our measure of disconnect, this implies η_t fell substantially in late February and early March but then increased and stayed high for the remainder of the sample period. In short, around this period the market experienced a regime transition from a connected regime characterized by high uncertainty and high volatility (HH) to a disconnected regime characterized by high uncertainty and low volatility (HL). In Figure 10 we collect the wealth evolution for all portfolio strategies together with a set of portfolio performance measures. The period considered in this analysis is from January 2020 until December 2020.

INSERT FIGURE 10 ABOUT HERE

The top two panels of Figure 10 show that the portfolio conditioning on both volatility and uncertainty performs well throughout the entire Covid-19 period. Panel A shows that, at the two-week horizon, besides protecting the investor's wealth at the onset of the crisis in March and April 2020, the strategy also took advantage of the subsequent recovery of equity markets by rapidly increasing its equity portfolio share. Accordingly, at this horizon the wealth level achieved by the portfolio conditioning on both volatility and uncertainty is the highest. Panel B shows that at the monthly horizon the wealth evolution of the passive index and the portfolio conditioning on both volatility and uncertainty are very similar. By contrast, for the same horizon, the portfolio conditioning on volatility only achieved the lowest wealth level, incurring cumulative losses by the end of 2020. Notably, the portfolio conditioning in both volatility and uncertainty was almost fully invested in the equity market, compared to only half of wealth invested in equities for the portfolio conditioning in volatility only. The high uncertainty around this period led to higher subsequent stock excess return, and only the portfolio conditioning in both volatility and uncertainty took advantage of this compensation for uncertainty, which the portfolio conditioned on volatility alone could not exploit.

Panels C and D show that the portfolio strategy conditioning on both volatility and uncertainty delivers the highest risk-adjusted return, as it achieves the highest Sharpe and Treynor ratios for most horizons considered (all except for the monthly horizon, in which conditioning on volatility and uncertainty delivers a risk-adjusted return similar to the one obtained by the passive index strategy). By contrast, the portfolio strategy conditioning in volatility only obtains the worst performance in most horizons, including negative Sharpe and Treynor ratios in some of them. Finally, panels E and F show that the active portfolio strategies preserve wealth more effectively than the passive index strategy, as they obtain lower maximum drawdowns and value-at-risk for most horizons (again all except for the monthly horizon, in which the three strategies achieve very similar wealth preservation). Meanwhile, both active portfolio strategies preserve wealth equally well, as none of them shows a lower maximum drawdown or value-at-risk for all horizons.

6. Out-of-sample analysis

In section 5, all the portfolio comparisons made were in-sample. It was a back-testing exercise, performed with the advantage of hindsight. In this section, we examine the same three prediction models for the equity premium, but out-of-sample. We show that adding uncertainty as a predictor of future stock excess returns improves forecast accuracy out-of-sample, compared to forecasts conditioning on volatility only, or based on the unconditional average stock excess return. Because in the context of our regression based portfolio analysis higher stock excess return forecast accuracy drives higher portfolio performance, the results we provide here support the notion that the higher portfolio performance from conditioning on both volatility and uncertainty shown in section 5 holds also out of sample.

We start by partitioning the full sample in half. The estimation sample consists on the first half of observations (from January 1986 until June 2003), leaving the rest of the sample for implementing the forecasts out-of-sample.[18] We run similar regressions to the one in Equation (27), with the stock excess return regressed on: (i) only an intercept (unconditional model), (ii) an intercept and volatility, and (iii) an intercept, volatility, and uncertainty. We run the regressions and make forecasts over horizons ranging from 1 to 12 weeks. Starting from using only the first half of observations in our full sample, we update our estimation one week at a time by using a rolling expanding window.

Using the estimated regression coefficients, we go on to predict out of sample the stock excess return over horizons $\tau = 1, ..., 12$ weeks in the remaining observations available (starting from the July 2003 - December 2020 period). To evaluate the forecast performance we compare the ex-ante forecast of stock excess return according to each of the three regression based models considered above with the corresponding ex-post realized values, by means of the root mean squared error (RMSE). Table 3 shows the results.

INSERT TABLE 3 ABOUT HERE

Each column in Table 3 reports the RMSE of a forecast of stock excess return for each regression model considered. Remarkably, the results show that the richer model (conditioning on volatility and uncertainty) is more accurate (lower RMSE) than conditioning only on volatility, or not conditioning at all, for all horizons considered except the one-week horizon. This establishes that the in-sample higher performance of the conditional model based on volatility and uncertainty was not due to overfitting in-sample.

Conditioning only on volatility does not achieve similar success. For horizons from 1 to 7 weeks, forecasting the stock excess return conditioning on volatility is more accurate than not conditioning at all. But for longer horizons from 8 to 12 weeks, not conditioning provides a better forecast than conditioning on volatility. By contrast, for all horizons considered conditioning on volatility and uncertainty leads to more accurate forecasts of the stock excess return than not conditioning.

Overall, our results clearly show that a forecast of stock excess return conditioning on both volatility and uncertainty is superior to both a forecast conditioning on volatility only and a forecast based on the unconditional mean. Because in the context of our regression based portfolio analysis higher stock excess return forecast accuracy drives higher portfolio performance, the results we provide in this section support the notion that the higher portfolio performance from conditioning on both volatility and uncertainty shown in Section 5 holds also out of sample.

[18]We replicated the analysis using for the estimation sample other fractions of the full sample, including 40%, 60%, 70%, and 80%. In all cases, the results were qualitatively the same.

7. Conclusion

In this paper, we develop a novel and tractable framework in which stochastic volatility and model uncertainty are both stochastic, are possibly disconnected, and affect the optimal consumption and portfolio choice of an investor who is averse to both. By allowing uncertainty and volatility to be disconnected, we show that we achieve a better in-sample description and better out-of-sample forecasts of the equity premium.

In particular, we show that equity returns respond negatively to contemporaneous volatility (consistent with the prevalence of the leverage effect in the data), but respond positively, and consistently across horizons, to the uncertainty component in the model. Our results therefore show that the equity premium appears to be earned for facing uncertainty, especially high uncertainty that is disconnected from low volatility, rather than for facing volatility as traditionally assumed.

This has important implications for optimal portfolio allocation and asset prices. The model predicts that periods of high uncertainty are followed by subsequently higher excess returns, which are even higher in disconnect regimes characterized by high uncertainty and simultaneously low volatility. Both are salient features of the data. Accordingly, a reference investor forecasting the stock excess return by conditioning on both volatility and uncertainty achieves superior portfolio performance, over and above the one achieved by conditioning in volatility only, or by not conditioning at all.

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Appendix: Proofs

A.1 Derivation of the relative entropy growth

Given the explicit expression for the measure change ϑ_t in Equation (7), the relative entropy growth is computed as follows

$$\begin{aligned}\mathcal{R}(\vartheta_t) &:= \frac{d}{ds}\bigg|_{s=0} \mathbb{E}_t^{\vartheta}\left[\log \vartheta_{t+s}\right] = \frac{d}{ds}\bigg|_{s=0} \mathbb{E}_t^{\vartheta}\left[\int_t^{t+s} h_u dW_u^S - \int_t^{t+s} \frac{h_u^2}{2} du\right] \\ &= \frac{d}{ds}\bigg|_{s=0} \mathbb{E}_t^{\vartheta}\left[\int_t^{t+s} h_u dW_u^{S,\vartheta} + \int_0^t \frac{h_u^2}{2} du\right] = \frac{d}{ds}\bigg|_{s=0} \int_t^{t+s} \frac{h_u^2}{2} du = \frac{h_t^2}{2} \qquad \text{(A.1)}\end{aligned}$$

where the first equality follows by definition, the second one from Girsanov's theorem, i.e. $W_t^S = \int_0^t h_u du + W_t^{S,\vartheta}$, the third one due to the $\mathcal{F}_t$-measurability of h_t and because $W_t^{S,\vartheta}$ is a standard Brownian motion with mean zero (this is a standard Itô integral), and the last one from an application of the Leibnitz rule of differentiation of integrals.[19]

A.2 Derivation of the robust HJB equation for general preferences

To find a solution of the problem in Equation (16) subject to the entropy growth constraint in Equation (17), we formulate it as a Lagrange optimization problem with inequality constraints. We first determine the optimal robust control policy h_t^*, by solving a constrained optimization problem. Denote by $\mathcal{L} = \mathcal{L}(C_t, \omega_t, h_t, \theta)$ the Lagrangian associated to the constrained optimization problem given in equations (16) and (17), and by θ the corresponding Lagrange multiplier of the entropy growth constraint in Equation (17). Then the first order optimality conditions associated to the constrained minimization in equations (16) and (17) are

$$\frac{\partial \mathcal{L}}{\partial h_t} = \frac{\partial V}{\partial X} X\omega_t v_t \sigma_S - \frac{\partial}{\partial h_t}\theta\left(\frac{1}{2}h^2 - \frac{1}{2}\epsilon^2 \mathcal{U}_t^2\right) = 0, \qquad \text{(A.2)}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{1}{2}h^2 - \frac{1}{2}\epsilon^2 \mathcal{U}_t^2 = 0 \qquad \text{(A.3)}$$

$$\theta \geq 0 \qquad \text{(A.4)}$$

$$\theta\left(\frac{1}{2}h^2 - \frac{1}{2}\epsilon^2 \mathcal{U}_t^2\right) = 0. \qquad \text{(A.5)}$$

[19] Leibniz rule states that for a continuous function f, we have

$$\frac{d}{du}\left(\int_{a(u)}^{b(u)} f(x,u)\,dx\right) = \int_{a(u)}^{b(u)} \frac{\partial f}{\partial u}\,dx + f\big(b(u),u\big)\cdot b'(u) - f\big(a(u),u\big)\cdot a'(u)$$

where the functions $a(u)$ and $b(u)$ are both continuous and both have continuous derivatives in u.

The optimal drift perturbation component h_t^* is obtained from the solution to the system of equations (A.2) through (A.5), and is given by:[20]

$$h_t^* = -\epsilon \mathcal{U}_t = -\epsilon \eta_t v t. \tag{A.6}$$

We then plug this solution h_t^* back into the Lagrangian and solve the remaining maximization problem in Equation (16). The corresponding first order conditions associated to the investor's optimal consumption and portfolio policies are

$$\frac{\partial \mathcal{L}(h_t^*, \theta^*)}{\partial C_t} = \frac{\partial U(C_t)}{\partial C_t} - \frac{\partial V}{\partial X} = 0 \tag{A.7}$$

$$\begin{aligned} \frac{\partial \mathcal{L}(h_t^*, \theta^*)}{\partial \omega_t} &= \frac{\partial V}{\partial X} X_t \left(\mu_{S,t} - r_{f,t} + v_t h_t^* \sigma_S\right) + \frac{\partial^2 V}{\partial X^2} v_t^2 X_t^2 \sigma_S^2 \omega_t \\ &+ \frac{\partial^2 V}{\partial X \partial v_t} X_t \rho_{Sv} \sigma_S \sigma_v v_t^2 + \frac{\partial^2 V}{\partial X \partial \eta} X_t \rho_{S\eta} \sigma_S \sigma_\eta \eta_t v_t = 0, \end{aligned} \tag{A.8}$$

with corresponding optimal policies given by

$$C_t^* = \mathcal{I}\left(\frac{\partial V}{\partial X}\right), \tag{A.9}$$

$$\omega_t = -\frac{\frac{\partial V}{\partial X}\left(\mu_{S,t} - r_{f,t} + v_t h_t^* \sigma_S\right) + \frac{\partial^2 V}{\partial X \partial v_t} \rho_{Sv} \sigma_S \sigma_v v_t^2 + \frac{\partial^2 V}{\partial X \partial \eta} \rho_{S\eta} \sigma_S \sigma_\eta \eta_t v_t}{\frac{\partial^2 V}{\partial X^2} v_t^2 X_t \sigma_S^2}, \tag{A.10}$$

where the function $\mathcal{I}(y)$ is the inverse of the marginal utility function $\mathcal{I}(y) = \left(\frac{\partial U}{\partial X}\right)^{-1}$ and with h_t^* given in Equation A.6.

A.3 Proof of Proposition 1

We now specialize to the specific case of logarithmic preferences, $U(C_t) = \log C_t$, with inverse marginal utility function $\mathcal{I}(y) = \frac{1}{y}$. We conjecture a solution of the form:[21]

$$L(X, \eta, v) = \frac{\log(X)}{\beta} + \phi(\eta, v). \tag{A.11}$$

Plugging these functional forms into the optimal policies for drift perturbation, consumption, and portfolio in equations (A.6), (A.9), and (A.10), respectively, lead to the expressions in the proposition.

[20] While in principle there are two roots for the solution of $h_t^* = \pm\epsilon\mathcal{U}_t$, we focus on the realistic solution with $h_t^* = -\epsilon\mathcal{U}_t < 0$, which prevails when the expected return of the risky asset under the reference measure is positive.

[21] It can be verified that this conjecture provides a solution to the problem, with $\phi(\eta, v)$ satisfying an ordinary differential equation that may be solved in closed form depending on the assumed functional form of $\mu_{S,t}$ and $r_{f,t}$.

A.4 Proof of Proposition 2

The equity premium in Equation (24) follows from combining the optimal portfolio policy in Equation (20) with the market clearing condition for the stock in Equation (22). The price-dividend ratio given in the proposition follows from combining the optimal consumption policy in Equation (18) with the market clearing condition for consumption in Equation (21). In addition, one needs to take into account that because $C_t = D_t \ \forall t$, the price of a claim on the future stream of consumption must be equal to the price of a claim on the future stream of dividends, which implies that $X_t = S_t$. Because the price-dividend ratio is constant, the dynamics of the stock price and dividends are related by

$$\begin{aligned} \frac{dS_t}{S_t} &= (\mu_{S,t} - \beta)\, dt + \sigma_S v_t dW_t^S \\ &= \frac{dD_t}{D_t} = \mu_D dt + \sigma_D v_t dW_t^D, \end{aligned} \tag{A.12}$$

and identifying terms we get

$$\mu_{S,t} = \beta + \mu_D \text{ and } \sigma_S = \sigma_D. \tag{A.13}$$

The risk-free rate in Equation (25) follows from combining the equity premium in Equation (24) and the stock return drift and volatility in Equation (A.13).

	Full Sample	Financial Crisis	US 2016 election	Covid-19
Time Period Begin	Jan. 1986	Jul. 2007	Jul. 2016	Jan. 2020
Time Period End	Dec. 2020	Mar. 2009	Jan. 2018	Dec. 2020
Average stock excess return (%)	7.75	-35.42	14.52	16.80
Volatility of stock excess return (%)	21.01	25.48	10.49	25.65
Average risk-free rate (%)	3.22	1.98	0.69	0.52
Volatility of risk-free rate (%)	1.22	0.45	0.11	0.19
Average disconnect	1.00	0.59	1.42	1.49

Table 1: Moments of stock market return, risk-free rate, and disconnect

Notes: This table presents annualized first and second moments (in percentual terms) of the stock market excess return and the risk-free rate over the full sample and three sub-periods of interest. It also reports average disconnect (η_t) for the same periods. All data are nominal and sampled at the weekly frequency.

		Volatility Level	
		High	Low
Uncertainty Level	High	$\bar{r}_e = 23.17\%,\ \bar{r}_f = 1.29\%$ $\mathbb{P}[\text{HH}] = 9.23\%$	$\bar{r}_e = 26.33\%,\ \bar{r}_f = 2.39\%$ $\mathbb{P}[\text{HL}] = 10.82\%$
	Low	$\bar{r}_e = 5.56\%,\ \bar{r}_f = 3.1\%$ $\mathbb{P}[\text{LH}] = 10.36\%$	$\bar{r}_e = 3.07\%,\ \bar{r}_f = 3.62\%$ $\mathbb{P}[\text{LL}] = 69.59\%$

Table 2: Stock excess return and risk-free rate in different uncertainty and volatility regimes.

Notes: This table reports the estimated unconditional probability of each market regime (HH, HL, LH, and LL). It also shows the average (annualized) risk-free rate and stock excess return at a 2-weeks horizon. The threshold values for volatility and uncertainty are given by their mean plus one half of their standard deviation. We say that volatility and uncertainty are high (low), when they are above (below) their threshold values. Both volatility and uncertainty are sampled at the weekly frequency from January 1986 until December 2020.

τ	Unconditional	Volatility	Volatility and Uncertainty
1	1.558	1.552	1.557
2	1.989	1.986	1.978
3	2.277	2.274	2.258
4	2.527	2.522	2.503
5	2.754	2.750	2.725
6	2.950	2.946	2.915
7	3.130	3.126	3.088
8	3.288	3.289	3.241
9	3.438	3.444	3.389
10	3.590	3.595	3.538
11	3.743	3.762	3.703
12	3.888	3.932	3.870

Table 3: Root mean squared error ($\times 100$) for different out-of-sample stock excess return forecasts.

Notes: This table reports the root mean squared error (RMSE), multiplied by 100 for ease of illustration purposes, from out-of-sample forecasts of stock excess return using horizons 1 through 12 weeks, based on three different models: (i) $\widehat{r}_{e,t+\tau} = \widehat{\beta}_0$ (unconditional mean equal to 7.55%), (ii) $\widehat{r}_{e,t+\tau} = \widehat{\beta}_0 + \widehat{\beta}_v v_t^2$ (conditioning on volatility), and (iii) $\widehat{r}_{e,t+\tau} = \widehat{\beta}_0 + \widehat{\beta}_v v_t^2 + \widehat{\beta}_\eta \eta_t v_t^2$ (conditioning on volatility and uncertainty). Lower RMSE indicate a better forecast performance. The estimation period starts by using the first half of the full sample: from January 1986 until June 2003. We then update our estimation one week at a time by using a rolling expanding window. The out-of-sample period corresponds to the remaining observations in our sample (starting from the July 2003 - December 2020 period).

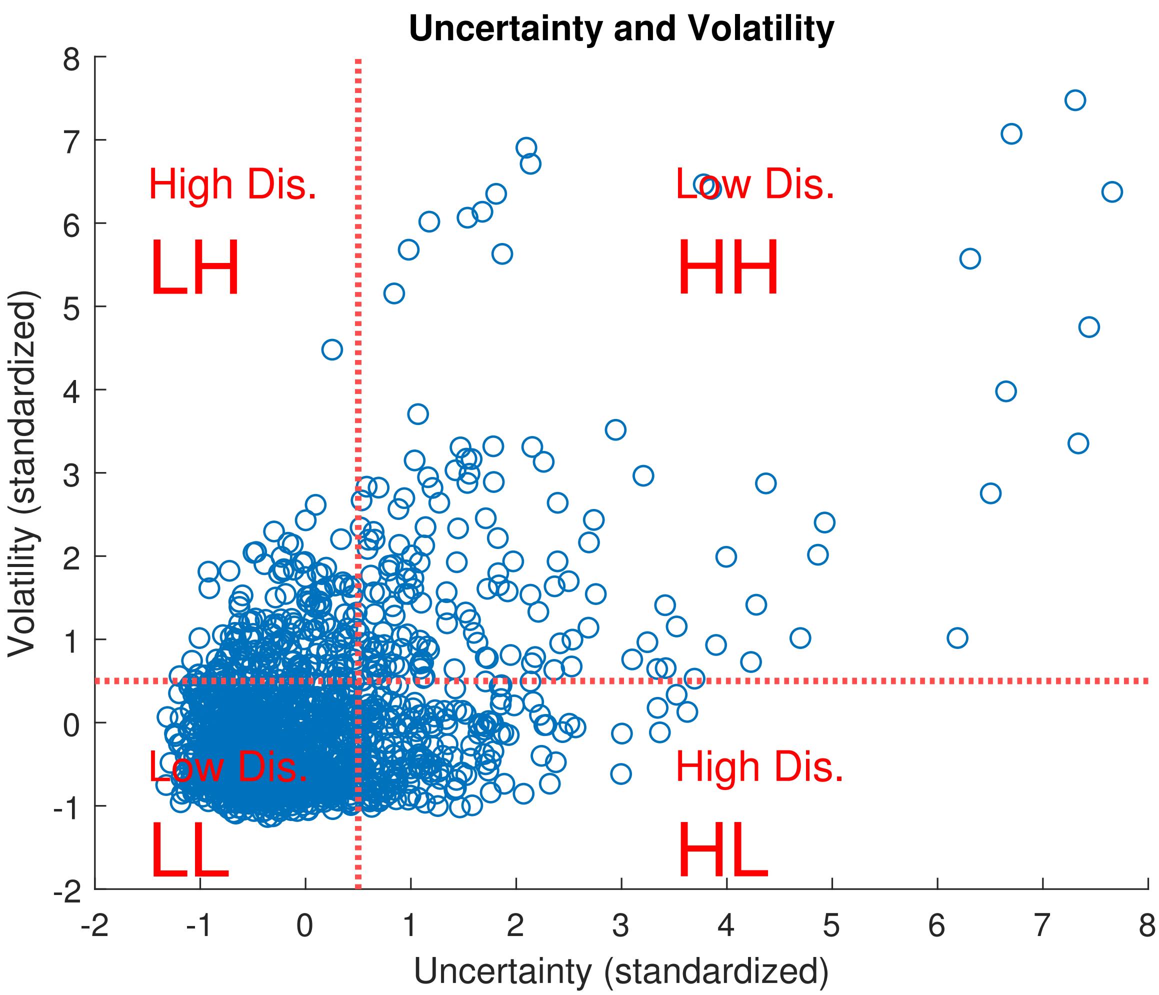

Figure 1: Uncertainty and volatility regimes

Notes: The figure shows a scatter plot of standardized uncertainty (proxied by the economic policy uncertainty index EPU_t) and volatility (proxied by realized volatility). Both are sampled at the weekly frequency from January 1986 until December 2020. High disconnect occurs when either uncertainty is high while volatility is low (denoted "HL" and corresponding to $\eta_t > 1$) or when uncertainty is low while volatility is high (denoted "LH" and corresponding to $\eta_t < 1$). In the other two quadrants, uncertainty and volatility are in sync and disconnect is low (corresponding to η_t close to 1). "High" are levels above the mean plus one half the standard deviation, and "low" are those below high levels.

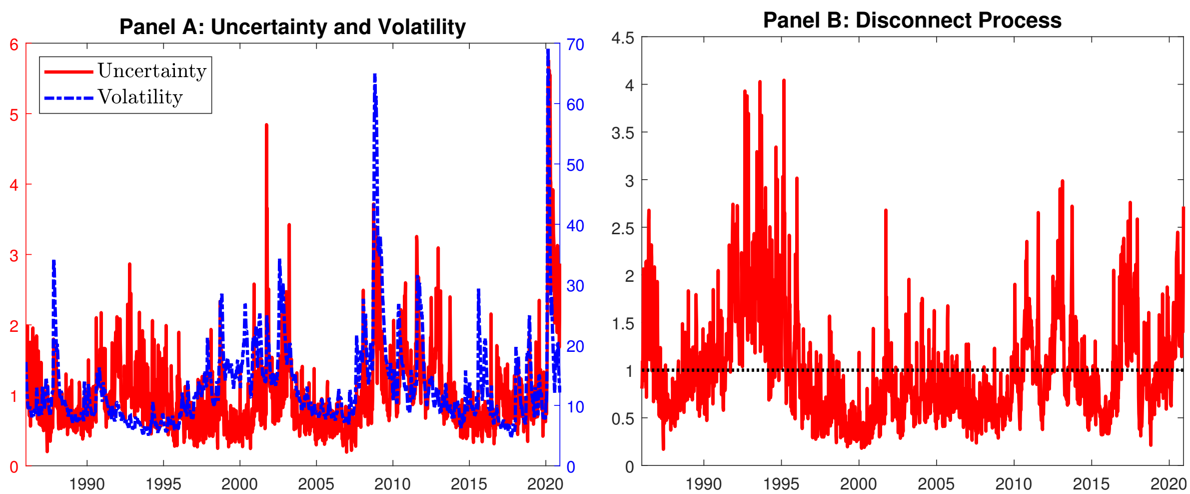

Figure 2: Time series of volatility, uncertainty and disconnect

Notes: Panel A shows uncertainty (proxied by the economic policy uncertainty index EPU_t scaled by 1/100) and volatility (proxied by realized volatility). Panel B shows the disconnect process (η_t) defined as the ratio between the five-day moving average of the EPU index divided by realized volatility, normalized to have a mean of 1. The sample period is from January 1986 until December 2020, and the data is sampled at the weekly frequency.

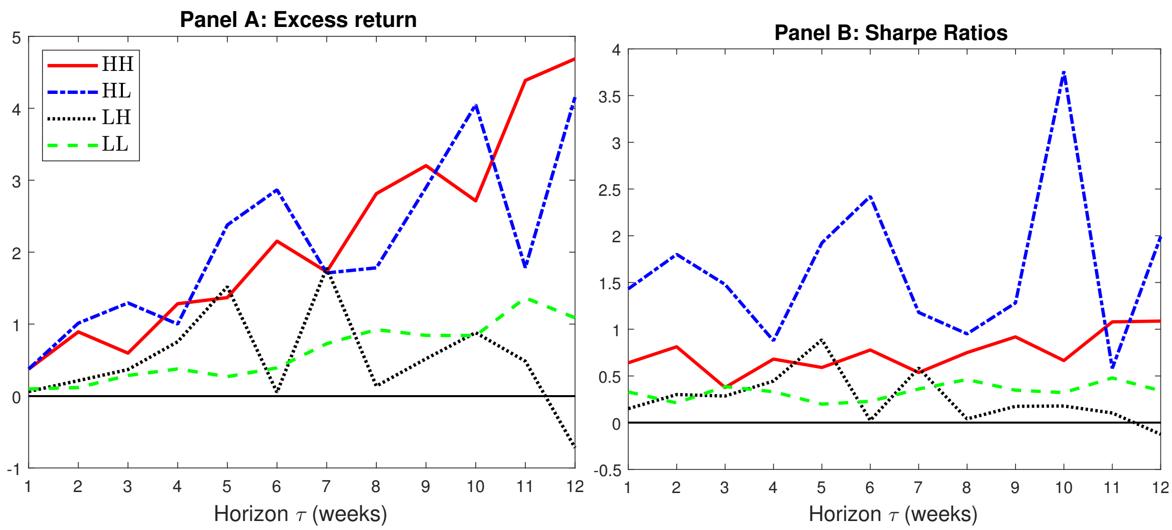

Figure 3: Realized stock excess returns and Sharpe ratios conditional on regimes of uncertainty and volatility

Notes: This figure shows annualized average stock excess returns (in %) and Sharpe ratios over a given horizon $\tau = 1, \dots, 12$ weeks, conditional on being in one of the four market regimes: (i) both volatility and uncertainty are high (HH), (ii) high uncertainty and low volatility (HL), (iii) low uncertainty and high volatility (LH), or (iv) both low uncertainty and low volatility (LL). The HH and LL regimes correspond to uncertainty and volatility being connected, and the HL and LH regimes are periods in which volatility and uncertainty are disconnected. Both volatility and uncertainty are standardized. "High" are levels above the mean plus one half times the standard deviation, and "low" are those below high levels. The sample period is from January 1986 until December 2020, and the data are sampled at the weekly frequency.

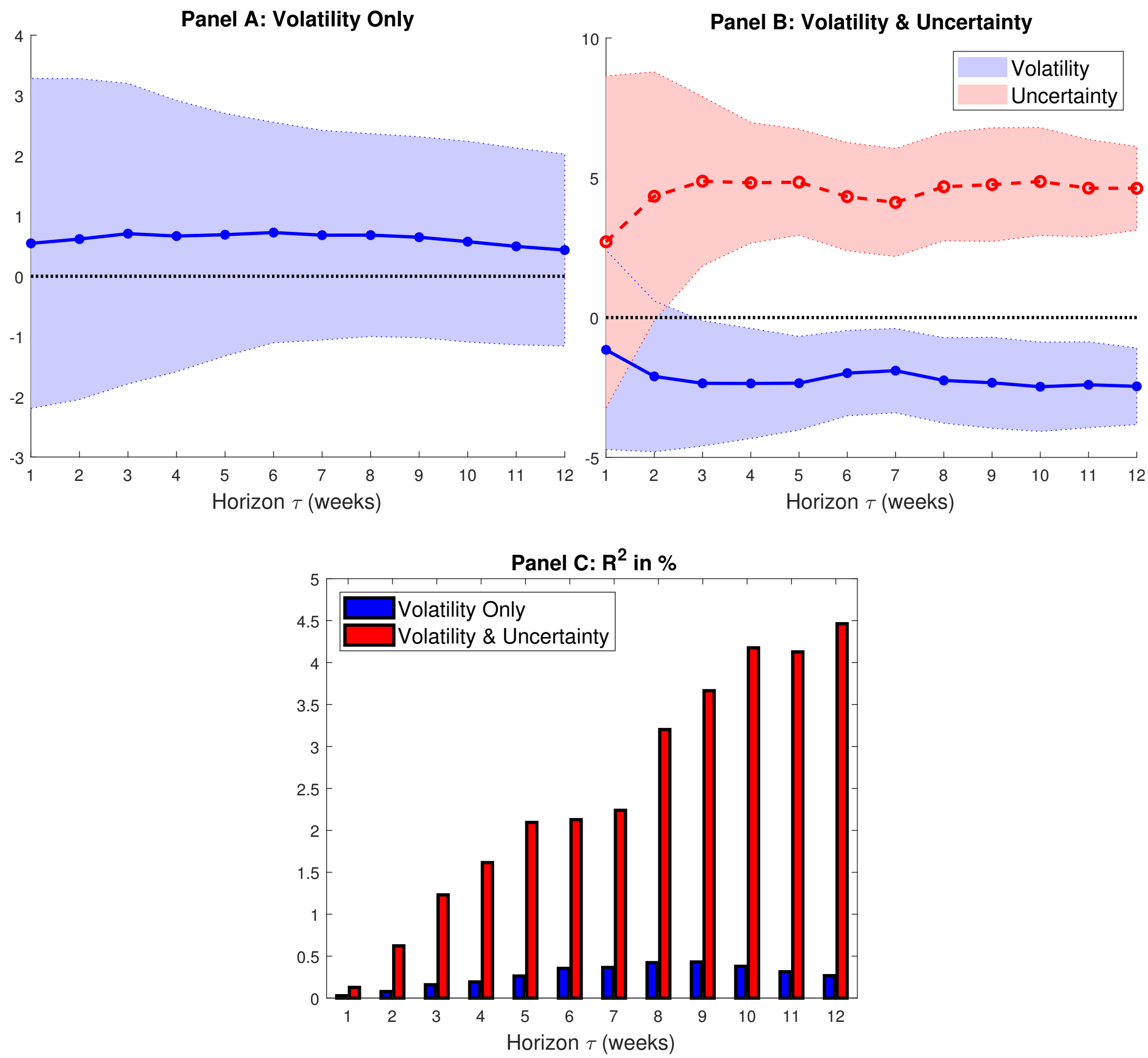

Figure 4: Regression of stock excess return on volatility and uncertainty

Note: The top charts in the figure show estimated regression coefficients (solid blue and dashed red lines) from two time-series regressions. Panel A corresponds to a uni-variate regression of stock excess returns over horizons of up to twelve weeks onto a variance term v_t^2. Panel B corresponds to a bi-variate regression of excess returns over horizons of up to 12 weeks onto a variance term given by v_t^2 and an uncertainty term given by $\eta_t v_t^2$. The form of the regression is implied by the equilibrium form of the equity premium in the model as derived in Proposition 2. The shaded areas represent 95%-confidence intervals constructed using auto-correlation and heteroskedasticity corrected standard errors (HAC). Panel C shows the R^2 extracted from the uni- and bi-variate regressions in Panels A and B. The sample period is from January 1986 until December 2020, and the data are sampled at the weekly frequency.

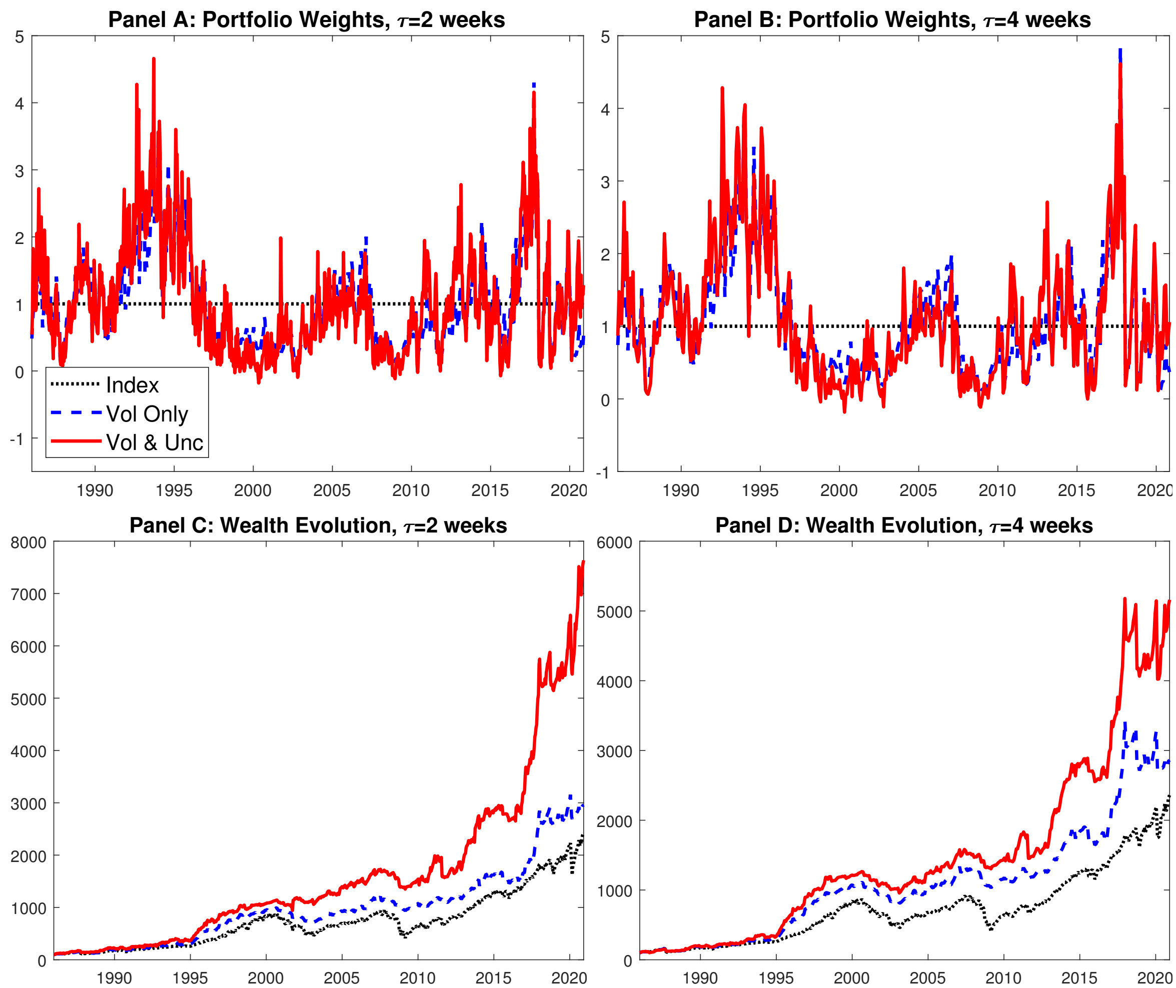

Figure 5: Portfolios and wealth evolution: full sample

Notes: Panels A and B show for two- and four-weeks horizons, respectively, the equity portfolio weights according to three strategies: "Index" consists on investing only in the stock ($\omega_t = 1$), "Volatility only" forecasts the equity premium conditioning on volatility only, and "Volatility and Uncertainty" forecasts the equity premium conditioning on both volatility and uncertainty. Panels C and D show for two- and four-weeks horizons, respectively, the cumulative wealth evolution associated with each portfolio strategy in Panels A and B. The initial wealth in panels C and D is fixed to 100. The investment and model estimation periods are both from January 1986 until December 2020 (full sample).

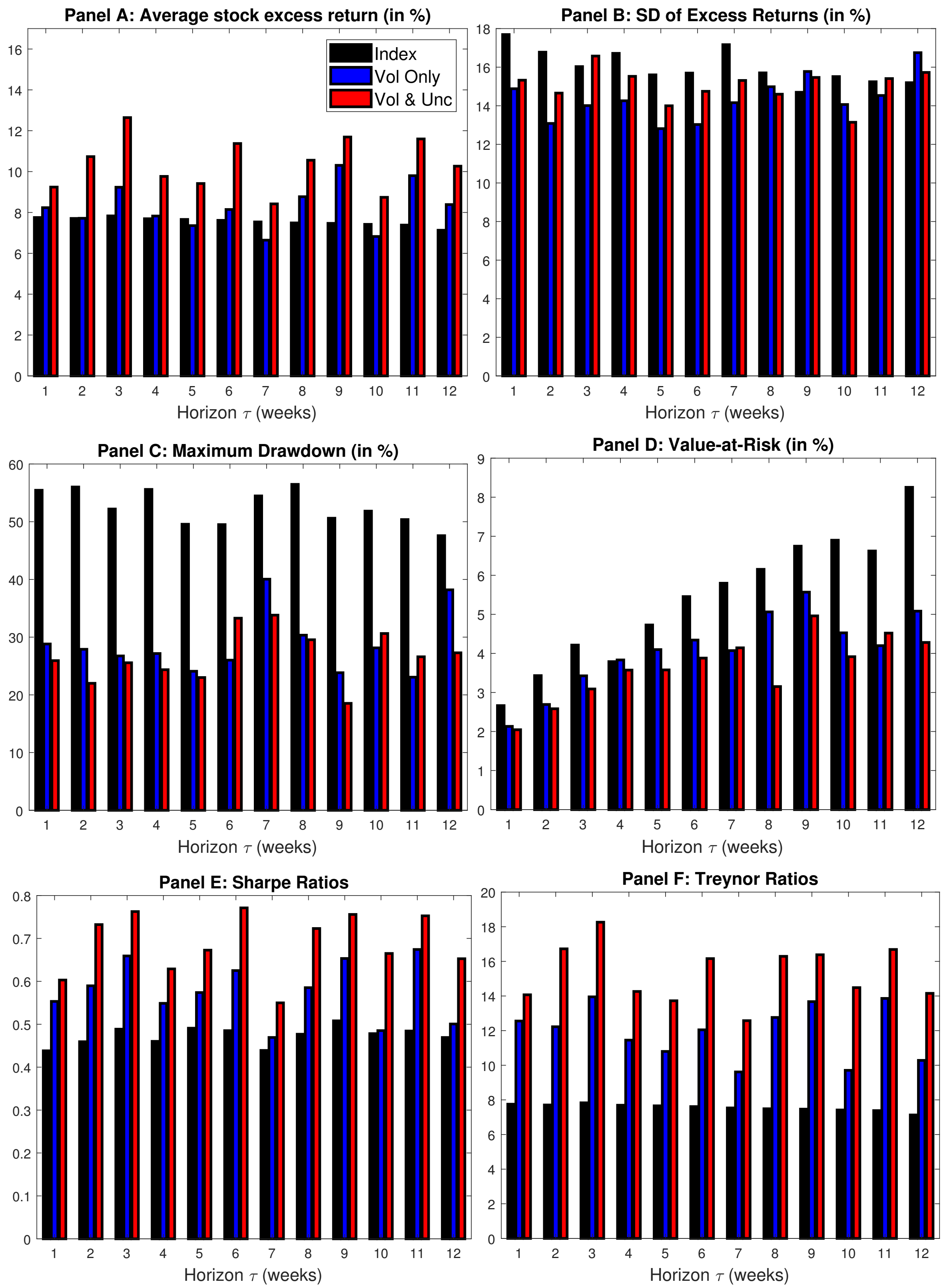

Figure 6: Portfolio performance measures: full sample

Notes: This figure reports measures of risk-adjusted wealth excess returns for: index investing, portfolio conditioning on volatility only, and portfolio conditioning on volatility and uncertainty. The investment and model estimation are done over the full sample.

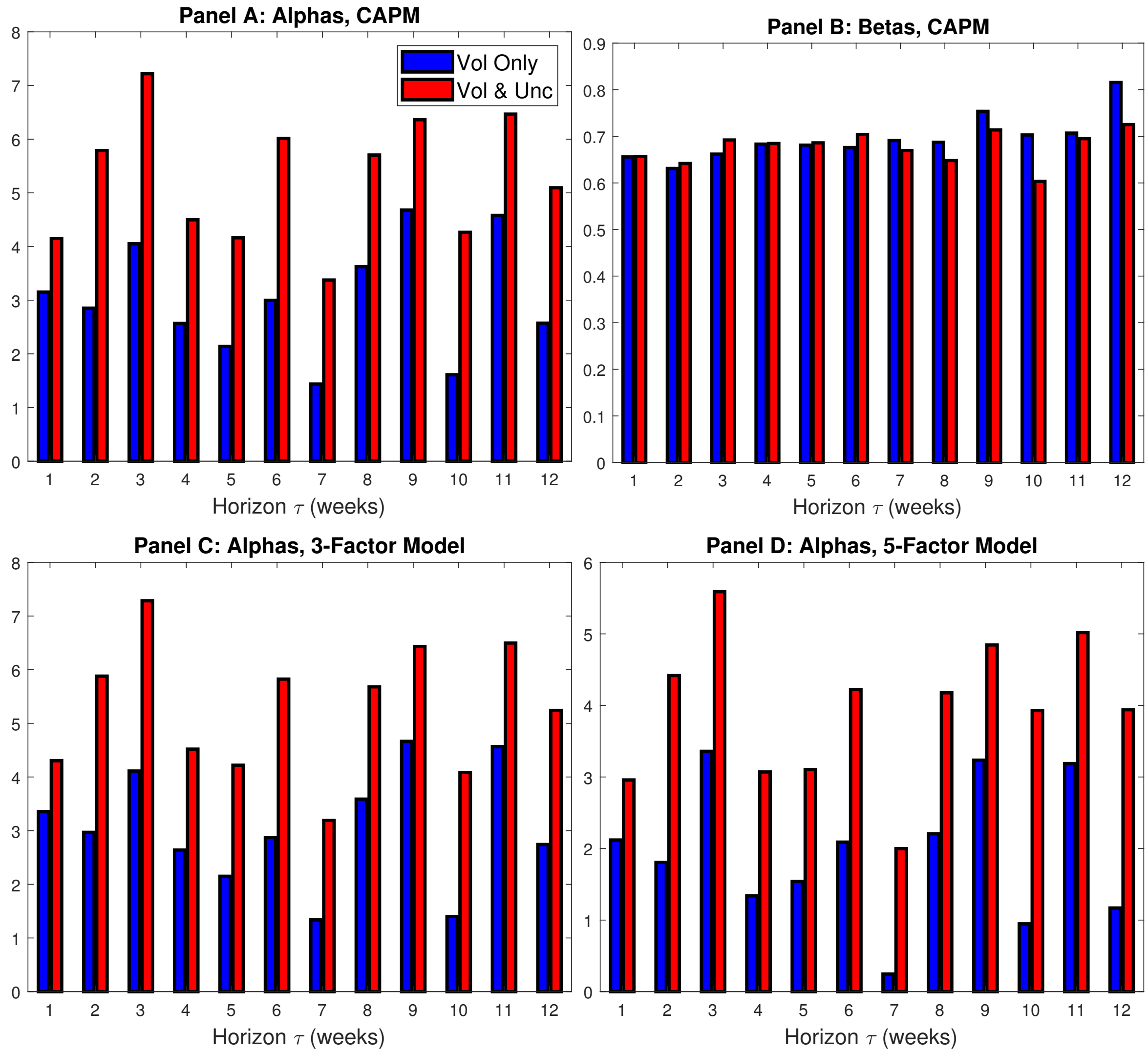

Figure 7: Alphas and betas: full sample

Notes: Panels A and B show the estimated alpha and beta from a CAPM model, respectively. These are obtained by regressing the excess return of wealth associated with each active portfolio strategy onto the excess return of the equity market. Panels C and D show estimated alphas from alternative models. The three-factor alpha is obtained from performing the regressions onto the excess return of the market, the SMB ("Small Minus Big") factor, and the HML ("High Minus Low") factor. The five factor alpha is obtained from regressing onto the excess return of the market, the SMB factor, the HML factor, the RMW ("Robust Minus Weak") factor, and the CMA ("Conservative Minus Aggressive") factor. The investment and model estimation periods are both from January 1986 until December 2020.

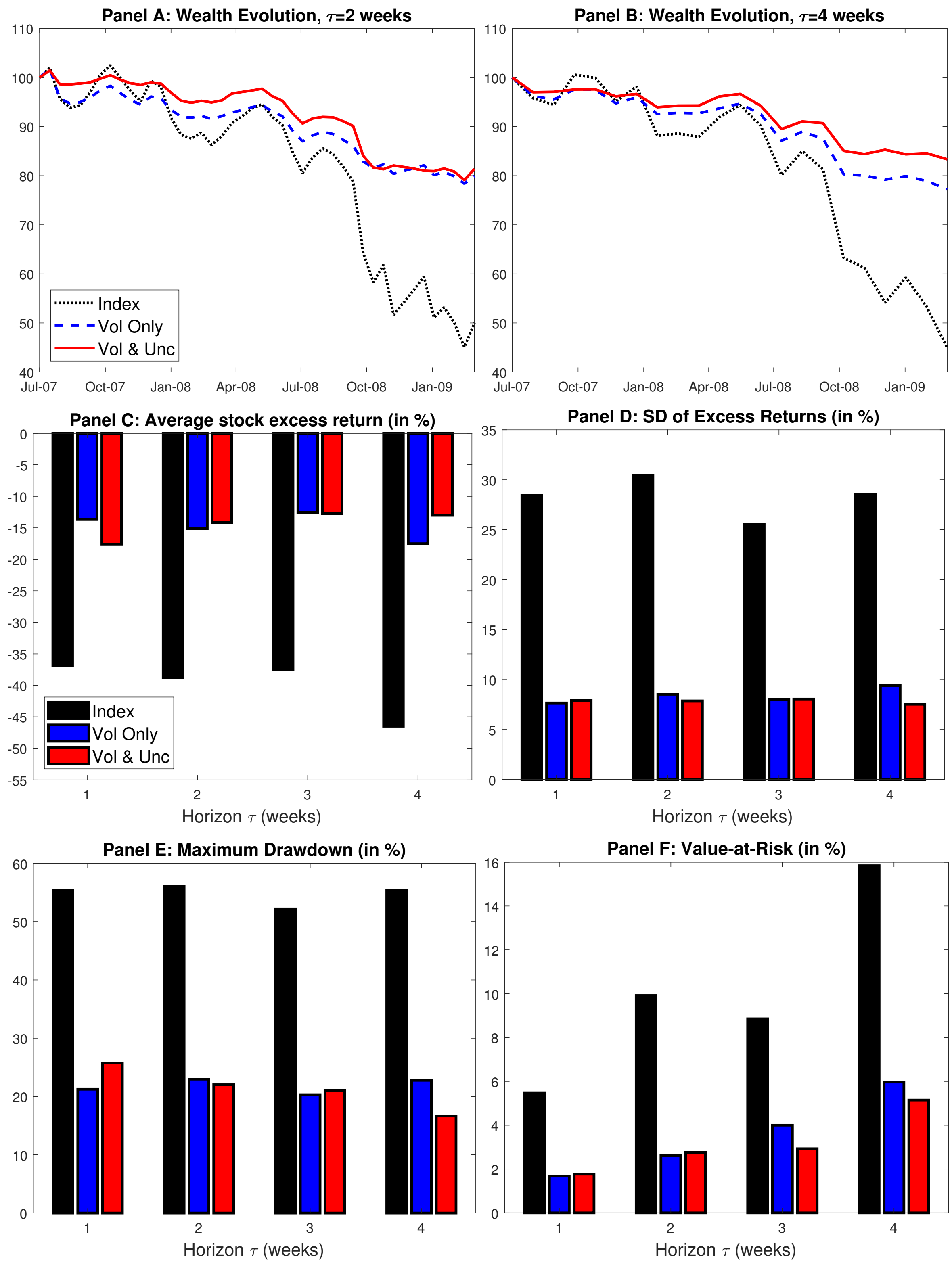

Figure 8: Wealth evolution and portfolio performance measures around the 2008 Financial Crisis

Notes: The elements in this figure match closely those from figures 5 and 6, except that this figure is restricted to the financial crisis period (from July 2007 until March 2009). The parameters of the model are estimated over the full sample (from January 1986 until December 2020). The initial wealth in panels A and B is fixed to 100.

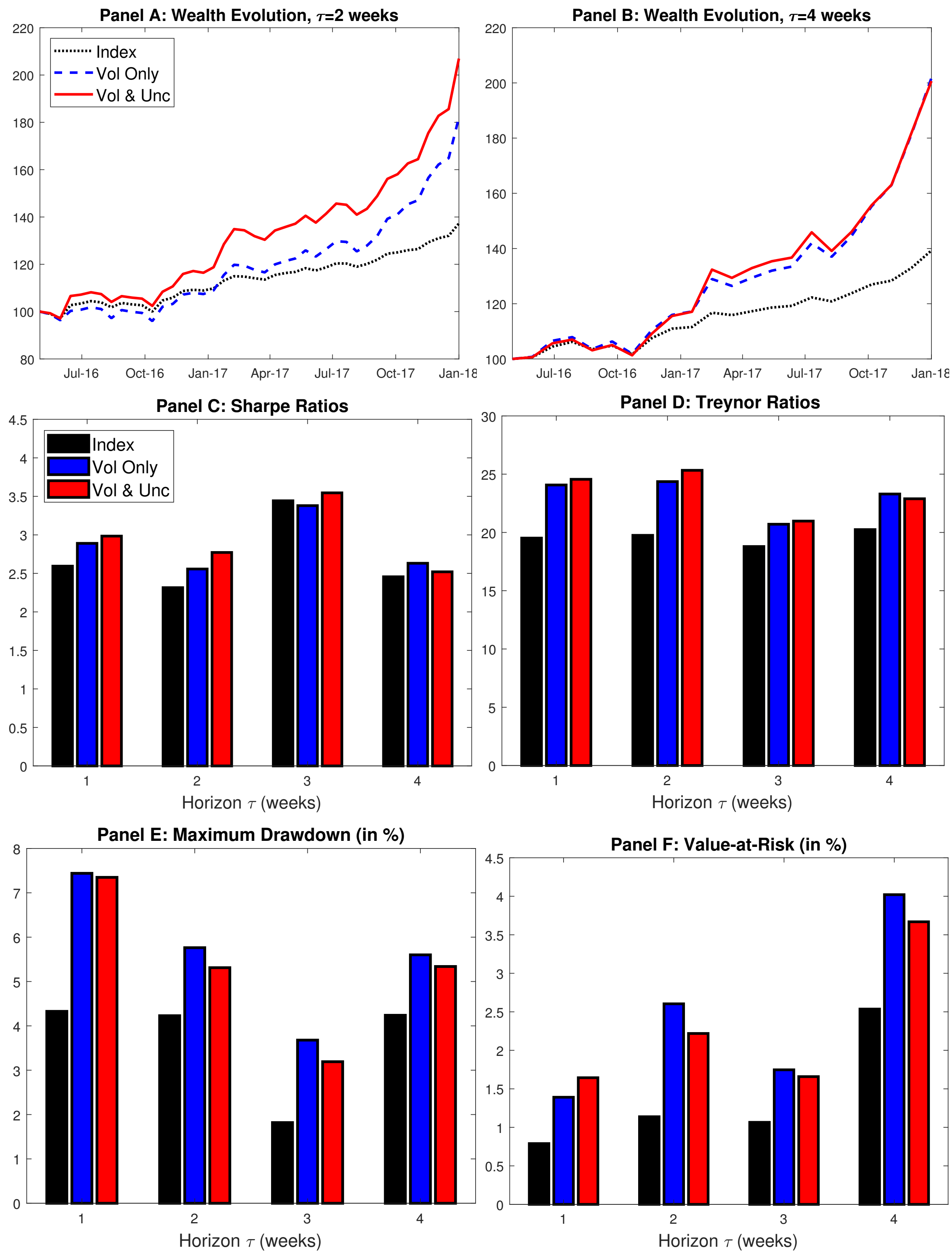

Figure 9: Wealth evolution and portfolio performance measures around the 2016 US Election

Notes: The elements in this figure match closely those from figures 5 and 6, except that this figure is restricted to the 2016 US election period (from July 2016 until January 2018). The parameters of the model are estimated over the full sample (from January 1986 until December 2020). The initial wealth in panels A and B is fixed to 100.

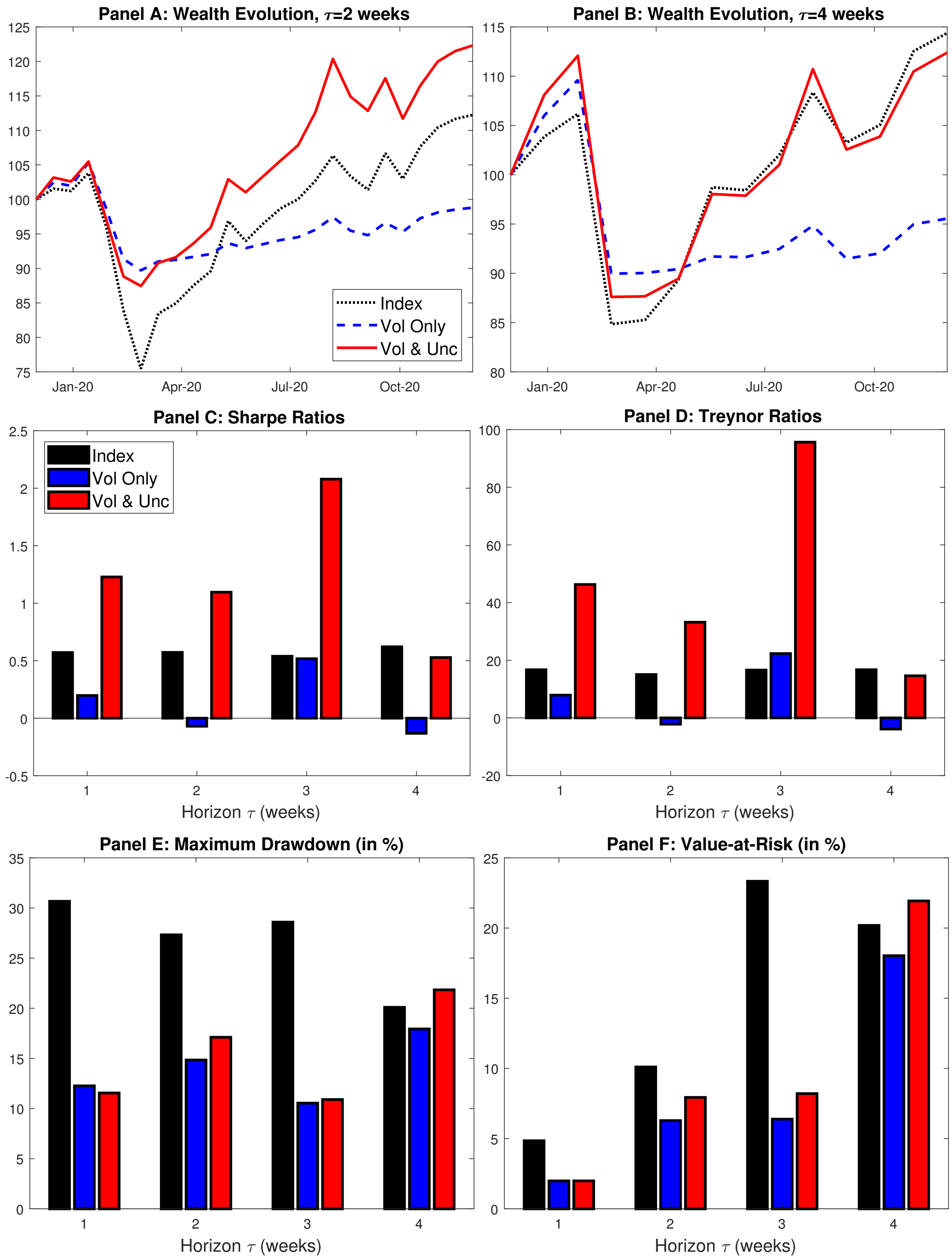

Figure 10: Wealth evolution and portfolio performance measures around the COVID-19 pandemic

Notes: The elements in this figure match closely those from figures 5 and 6, except that this figure is restricted to the COVID-19 pandemic period (from January 2020 until December 2020). The parameters of the model are estimated over the full sample (from January 1986 until December 2020). The initial wealth in panels A and B is fixed to 100.