Trend-following Hedge Funds
and
Multi-period Asset Allocation

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Abstract

Selected hedge funds employ trend-following strategies in an attempt to achieve superior risk adjusted returns. We employ a lookback straddle approach for evaluating the return characteristics of a trend following strategy. The strategies can improve investor performance in the context of a multi-period dynamic portfolio model. The gains are achieved by taking advantage of the funds' high level of volatility. A set of empirical results confirms the advantages of the lookback straddle for investors at the top end of the multi-period efficient frontier.

1. Introduction

Over the past few years, hedge funds and related alternative investments have gained in popularity as traditional assets such as equities, corporate bonds, and venture capital have lost considerable market value due to the worsening economic conditions. Ideally, a hedge fund offers diversification benefits without a large reduction in expected profit/returns. University endowments in the US have benefited by this approach (see Swensen 1999). In this context, a desirable pattern would be to achieve the expected gains of equity assets or even better, with substantial diversification benefits. While this goal may be elusive in the short run, we show that certain forms of hedge funds – namely funds that follow trends – can play a role in dynamic, multi-period asset allocation.

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Within the alternative asset universe, hedge funds are increasingly popular as possible investment opportunities for wealthy investors who possess long time horizons. It is therefore of interest to study the problem of how this alternative asset class modifies such investors' allocation problem. In this article, we employ a straightforward model of a certain type of hedge fund's returns, namely a primitive trend-following strategy described by a lookback straddle. Fung and Hsieh (1997) introduced this model; we describe how these funds capture long volatility positions and are thereby desirable for multi-period investors.

Private investment vehicles, such as hedge funds, private equity and venture capital, have grown in size and importance over the past twenty years. A characterization of hedge funds (and similar commodity trading advisors (CTAs)) is given by Fung and Hsieh (1999). We consider only those hedge funds that are broadly described as trend following or market timing. According to Table 5 of Fung and Hsieh (1999), 58.1% of CTAs pursue trend following strategies. This may also be the case for a much smaller fraction of general hedge funds. However, Fung and Hsieh (2001) discuss the connection between market timing, as described by Merton (1981), and trend following, so the present analysis may apply to a larger segment of the hedge fund world, namely market timers.

Fung and Hsieh analyzed the returns of many hedge funds and their empirical relationships to a number of asset classes and concluded that the primitive trend following properties of many of these funds was reasonably modeled by replicating a lookback straddle on equities. They first used an extension of Sharpe's (1992) asset class factor model for style analysis to illustrate that dynamic, rather than buy-and-hold, trading strategies better explain hedge fund returns (Fung and Hsieh, 1997). Developing this idea further leads to the conclusion that selected hedge funds generate option-like returns. Specifically looking at trend-following funds, Fung and Hsieh (2001) suggest the option that best models these strategies is a lookback straddle. This is the model we shall use as a proxy for hedge funds.

The next section sets up the framework for the environment in which the quantitative analysis was done. We discuss constructing a lookback asset category and the multi-period stochastic optimization model. Results and conclusions are presented in Sections 3 and 4. We see that the lookback straddle benefits multi-period investors who have low levels of risk aversion.

2. Methods and Models

2.1: The Lookback Straddle

The lookback straddle is a derivative security that pays the holder the difference of the maximum and minimum prices of the underlying asset over a given time period. Suppose this time period is [0,T] and let (S_t) denote the price process of the underlying. Then a lookback straddle is simply a combination of a lookback put, which pays

$$S_{max} - S_T$$

and a lookback call which pays

$$S_T - S_{min} .$$

We shall refer to the straddle combination as a single derivative with payoff

$$S_{max} – S_{min} .$$

There is a closed form solution for the Black-Scholes *no-arbitrage* price of a lookback straddle. The method for obtaining this formula, which is convenient for the simulation in Section 3, is discussed in Goldman *et al*. (1979) and results in the following:

Define

S_t : The price of the underlying at time t;

J_t : The running maximum of S from time 0 up to time t;

I_t : The running minimum of S from time 0 up to time t.

That is,

$$I_t = \min_{0 \le u \le t} S_u \quad J_t = \max_{0 \le u \le t} S_u \ .$$

In addition, we define

r : The short rate or risk-free rate of return;

σ : The volatility of the underlying S;

N : The cumulative normal distribution function.

Then the formula for the price of the lookback call is:

$$C(t, S_t, I_t) = S \cdot \left(N(a_1) - \frac{\sigma^2}{2r} \cdot N(-a_1) \right) - I_t \cdot \left(e^{-r(T-t)} \cdot N(a_2) - \frac{\sigma^2}{2r} \cdot \left(\frac{I_t}{S} \right)^{\frac{r - \frac{\sigma^2}{2}}{2\sigma^2}} \cdot N(-a_3) \right), \quad (2.1)$$

where

$$a_1 = \frac{\log\left(\frac{S}{I_t}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot (T-t)}{\sigma \cdot \sqrt{(T-t)}} \quad a_2 = a_1 - \sigma \cdot \sqrt{(T-t)}$$

$$a_3 = \frac{\log\left(\frac{S}{I_t}\right) - \left(r - \frac{\sigma^2}{2}\right) \cdot (T-t)}{\sigma \cdot \sqrt{(T-t)}}$$

Similarly, the formula for the lookback put is:

$$P(t, S_t, J_t) = -S \cdot \left(N(b_2) - \frac{\sigma^2}{2r} \cdot N(-b_2) \right) + J_t \cdot \left(e^{-r(T-t)} \cdot N(b_1) - \frac{\sigma^2}{2r} \cdot \left(\frac{J_t}{S} \right)^{\frac{2r}{\sigma^2} - 1} \cdot N(-b_3) \right), (2.2)$$

where

$$b_1 = \frac{\log\left(\frac{J_t}{S}\right) - \left(r - \frac{\sigma^2}{2}\right)\cdot(T-t)}{\sigma \cdot \sqrt{(T-t)}} \quad b_2 = b_1 \text{-} \sigma \cdot \sqrt{(T-t)}$$

$$b_3 = \frac{\log\left(\frac{J_t}{S}\right) + \left(r - \frac{\sigma^2}{2}\right)\cdot(T-t)}{\sigma \cdot \sqrt{(T-t)}}$$

The price of the lookback straddle is given by:

$$\mathrm{L}(t, S_t, I_t, J_t) = P(t, S_t, J_t) + C(t, S_t, I_t)\ . \tag{2.3}$$

The general idea is that the holder benefits from the dominant trend in the underlying over the given time period. In particular, he or she profits from a large spread between the maximum and minimum, which is an indicator of high volatility. The lookback straddle serves as a condensation of hedge funds' strategies that seek to time the market and are neutral about its direction. Although there may be simpler trend following rules, the lookback straddle is a convenient idealization of the concept of capturing the dominant trend of the underlying. It is an extension of Merton's (1981) market timer model which was simply a straddle.

2.2: Market Model and Scenario Generation

To model future uncertainty of asset returns in our portfolio problem, we utilize a representative set of scenarios. Stochastic differential equations and time series analysis are two commonly used techniques to generate anticipatory scenarios. In this paper, we employ the former technique, in which a sequence of economic factors (for example GDP, corporate earnings, interest rates and inflation) drives the asset returns. The basic idea is to generate a scenario tree (Figure 2.2) for the asset returns over a pre-defined planning horizon. In our empirical tests, we discretize the planning period into 40 quarterly time periods. Each scenario depicts a single path in this scenario tree. A standard variance reduction method, antithetic variates, is employed to improve the accuracy of the model's recommendations. We employ indirect inference methods for calibrating the parameters of the resulting stochastic system (Gourieroux, Monfort and Renault 1993).

The expected annualized arithmetic and geometric returns and the expected annualized risks of each asset class in the opportunity set are shown in Table 3.1.

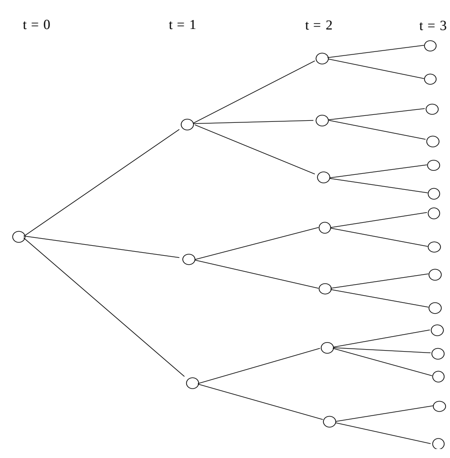

Figure 2.2: Standard tree representation for generating scenarios.

2.3: The Lookback Straddle Asset Class

The scenario generator simulates market conditions over a ten-year period at quarterly intervals. To describe hedge funds as alternative investments, we use a rolling portfolio of lookback straddles on US equities. There are many ways to construct this portfolio using options of different maturities each carrying different weights, which might be calibrated from real hedge fund returns. In this paper, we choose to use a simple strategy of buying one lookback straddle at time zero, holding it until expiration, then rolling the payoff into a new straddle of the same maturity, resulting in a self-financing portfolio. We study the properties of returns generated by this strategy using lookback straddles with maturities as short as one quarter (3 months) up to ten years.

The scenario generation produces quarterly returns, but a holder of a lookback straddle should obtain the difference between the maximum and minimum as if the underlying were monitored continuously. To track the maximum and the minimum on a daily basis, we locally approximate the intra-quarter trajectories using geometric Brownian motion with the overall ten-year historical volatility of the scenarios (23%), constructed with a Brownian bridge. Even though this reduces the monitoring bias, it does not eliminate it, and furthermore it introduces a new problem. That is, we are now simulating the underlying asset class with geometric Brownian motion even though the scenario generator uses a more complex and realistic model. We deal with the collective effect of these biases by pricing the lookback straddles, using the Black-Scholes formulas given above, with a *pricing volatility* that is lower than the historical volatility of the scenarios.

To find an asset class with the appropriate properties, we look at choosing different maturities and pricing volatilities. The statistics are given in Table A.1. There is a tradeoff between the pricing volatility and the trend-following horizon (or option maturity). An investor with a short investment horizon is looking for a significant trend to materialize in the near future, and thus his returns are very sensitive to the price of the lookbacks, as can be seen from Table A.1. Longer maturity investors' returns are less sensitive to the price.

In our analysis, we choose lookback straddles with one-year maturity priced at 14% volatility. This asset class has low geometric return (5%) and high volatility (59%) and captures the basic characteristics of the primitive trend following strategies (PTFS) used by Fung and Hsieh (2001, Table 1).

To summarize the characteristics of this asset class, we compute the geometrically averaged return over each ten-year scenario. We also constructed for comparison an alternative portfolio consisting of European straddles, which comprises a call and put both with at-the-money strike prices. This is Merton's market timing model. The histograms of the returns for the two investments are shown in Figure 2.3.

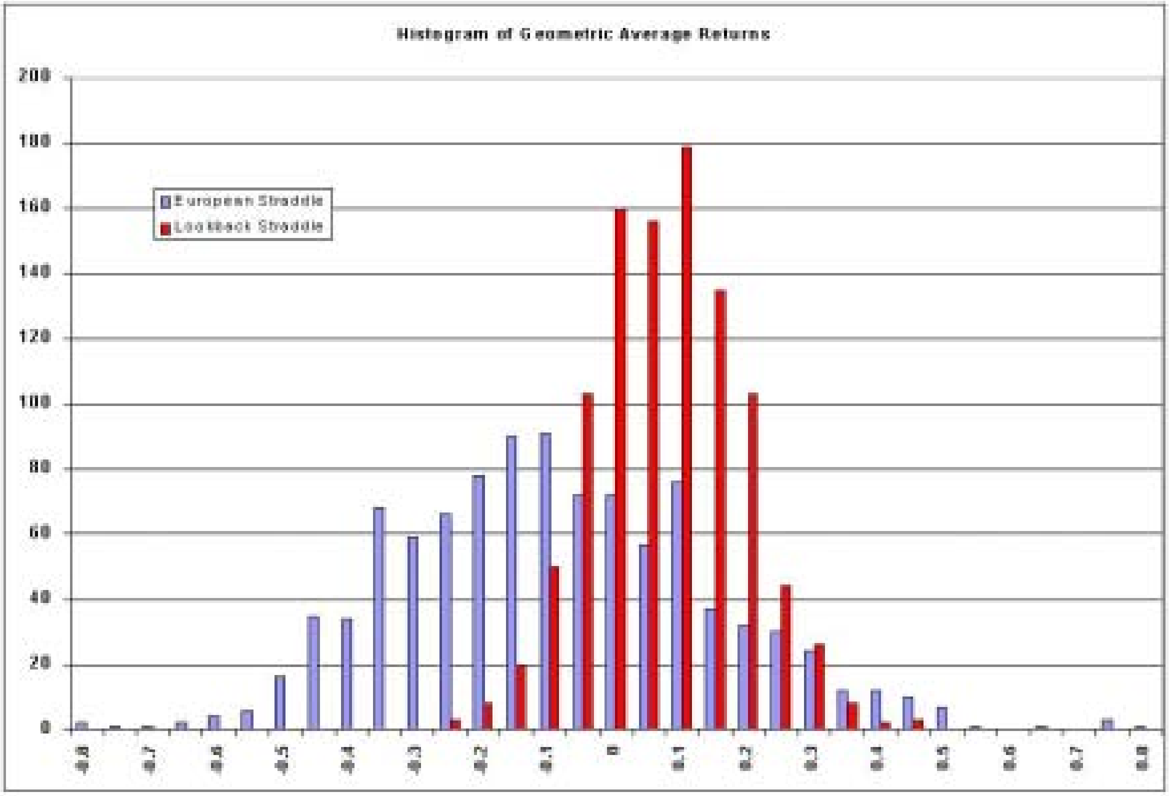

Figure 2.3: Return histograms for the different portfolios over the 1000 scenarios used in the simulations. Notice that the lookback straddles generate more positive returns with a narrower spread than the European straddles.

The mean geometric returns over the scenarios are given below:

Portfolio Type	Mean Geometric Return
European Straddle	-0.1134
Lookback Straddle	0.0506

This shows that the lookback straddle outperforms the European straddle, but its return is significantly lower over the ten-year period than the return on US equities, which is 9% in these simulated scenarios. The average volatilities are:

Portfolio Type	Average Volatility
European Straddle	141.83 %
Lookback Straddle	59.49%

Notice, the volatility of the lookbacks are lower than the European straddles, but they are still significantly higher than, for example, US equities (23%).

Finally we show in Figure 2.4, the sensitivity of the risk and return of the lookback asset class to small changes in the pricing volatility. Note that the risk is relatively insensitive, but that taking a pricing volatility 14.7% results in a negative geometric return. We discuss in Section 4 how asset classes with negative geometric returns would not improve the efficient frontier.

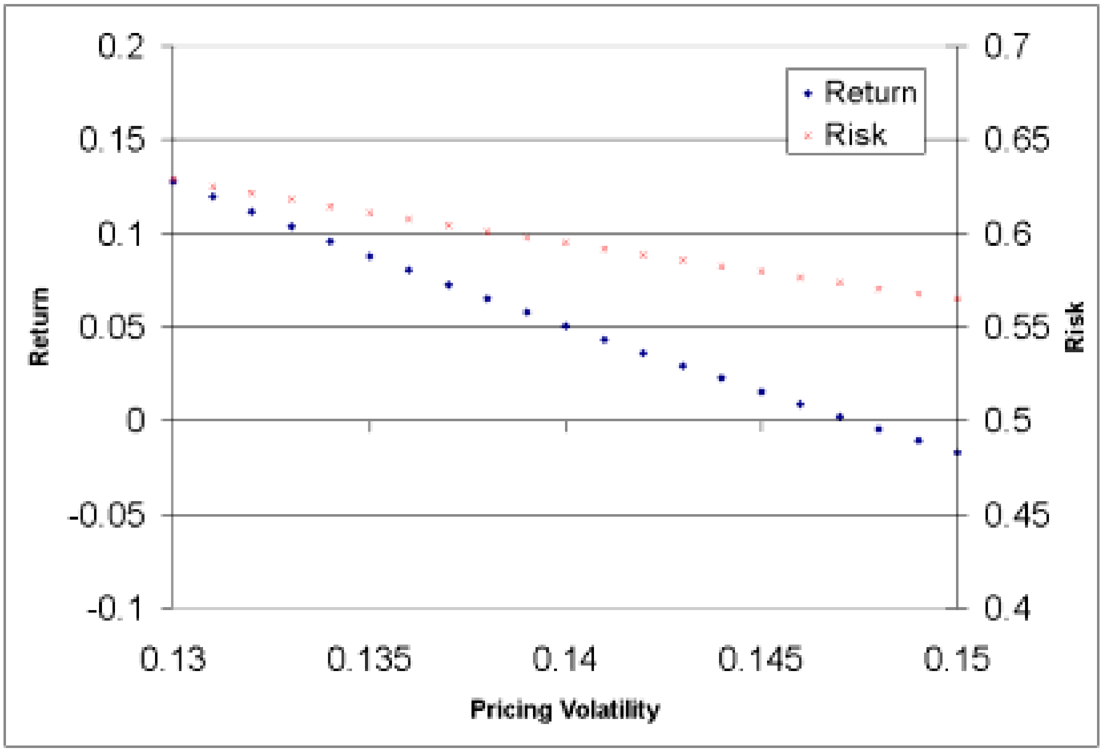

Figure 2.4: Sensitivity of lookback straddle asset class geometric returns and volatility to the pricing volatility.

On the face of it, even the lookback asset class that we choose, with high volatility and low (positive) return, also would not seem like a good investment by any reasonable optimization criteria. However, in a multi-period framework, there is opportunity for volatility pumping, as we will show in Section 3. That is, the real value to an investor of dynamic strategies such as lookback straddles is when they are incorporated within a portfolio model with dynamic rebalancing.

2.4: Stochastic Optimization Algorithm

In this section, our goal is to analyze the effects of adding lookback straddles as an alternative investment option into a portfolio consisting of the following asset classes: cash, US equity, European equity, Far East equity, Private equity, government bonds and

US high-quality bonds. For the purpose of simplicity, we make the following assumptions.

1. Transaction costs, market impact costs, and liquidity considerations are ignored.
2. Alternative investments are held through tax-exempt or tax-deferred locations.

In this paper we will stick with those assumptions but the model can be adjusted to incorporate taxes or transaction costs (see Kipcak 2001). Our analysis is presented in terms of efficient frontiers, plotting expected return versus risk.

2.5: Static Portfolio Model

First, we consider a static portfolio model, which is an application of the *buy-and-hold* decision rule over the 40 quarters. An investor who employs the *buy-and-hold* policy, as its name implies, decides upon a certain asset allocation to start with and does not trade at all until termination. This is the traditional Markowitz mean-variance framework. Note that short positions are not allowed. The algebraic formulation of the optimization model is as follows:

Parameters:

M : Number of scenarios (1000)

N : Number of asset classes (8)

T : Number of periods (40)

$r_{i,t}^{s}$: Rate of return for asset class i at time period t under scenario s

α : Reward coefficient ($0 \le \alpha \le 1$)

Variables:

x_i : Weight of asset i in the portfolio at the beginning of the investment horizon

R_t^s : Total return of the portfolio at time period t under scenario s

Model 1:

$$\max_{x_i} \alpha \frac{1}{M}\sum_{s=1}^{M}\sum_{i=1}^{N} x_i \prod_{t=1}^{T}\left(1+r_{i,t}^{s}\right) - (1-\alpha)\frac{1}{M}\sum_{s=1}^{M}\sqrt{4}\sum_{t=1}^{T}\frac{\left(R_t^s - E\left[R^s\right]\right)^2}{T-1} \qquad (2.4)$$

subject to

$$R_t^s = \frac{\sum_{i=1}^{N} x_i \prod_{j=1}^{t}\left(1+r_{i,j}^{s}\right)}{\sum_{i=1}^{N} x_i \prod_{j=1}^{t-1}\left(1+r_{i,j}^{s}\right)} \quad \text{for all } s=1..M \text{ and } t=2..T \qquad (2.5)$$

$$R_1^s = \sum_{i=1}^{N} x_i\left(1+r_{i,1}^{s}\right) \quad \text{for all } s=1..M \qquad (2.6)$$

$$\sum_{i=1}^{N} x_i = 1 \tag{2.7}$$

$$x_i \geq 0 \qquad \text{for all } i = 1..N \tag{2.8}$$

Equation (2.4) illustrates the bi-criteria objective function. The first term, weighed by α, depicts the expected final wealth after T quarters. The second term, whose weight is $(1-\alpha)$, is the expected annualized *temporal* variance of portfolio returns over the investment horizon. It acts as a penalization term. Equations (2.5) and (2.6) define the total portfolio returns per period under each scenario. Constraint (2.7) makes sure that the asset weights add up to 1. For convenience, the initial wealth is assumed to be 1 unit. Finally, constraint (2.8) assumes that short selling is not allowed.

In order to draw an efficient frontier, we solve the optimization problem using a non-convex solver (Maranas et al. 1997, and Mulvey and Ziemba 1998). Different points on the frontier are found by varying the value of α. Because we are interested in the effect of the allowing lookback straddles in our portfolio, we first restrict its weight to be 0 (eliminating it from the opportunity set). Then, we reintroduce it by removing this restriction.

2.6: Dynamic Portfolio Model

Next, we examine a dynamic portfolio model, which relies upon a *fixed-mix* decision rule. An investor who employs this fixed-mix policy sets a target asset allocation and maintains it throughout the planning horizon by trading at all available trading dates. As asset values move randomly, asset weights diverge from the target mix. The investor rebalances his portfolio to the target weights by transferring assets from one to another at the end of each trading date (i.e. every quarter, in our case). As is well known (Merton, 1990), many simple continuous-time investment models for utility maximization yield fixed-mix optimal strategies, which motivates these as a convenient basis for our dynamic portfolio model. See Mulvey *et al.* (1997) and Perold and Sharpe (1988) for further applications.

By omitting transaction costs, we eliminate the need to keep track of the weights before and after rebalancing. Instead, each period's portfolio return can be found simply by using the target mix and the individual asset returns for that period. Then, by accumulating the period returns, we will obtain the return over the investment horizon.

With the definition of the parameters and variables remaining the same, the dynamic portfolio model is formulated as follows:

<u>*Model 2:*</u>

$$\max_{x_i} \alpha \frac{1}{M} \sum_{s=1}^{M} \left(\left[\prod_{t=1}^{T} \sum_{i=1}^{N} x_i \left(1 + r_{i,t}^s\right) \right]^{4/T} - 1 \right) - (1-\alpha) \frac{1}{M} \sum_{s=1}^{M} \sqrt{4} \sum_{t=1}^{T} \frac{\left(R_t^s - E\left[R^s\right]\right)^2}{T-1} \tag{2.9}$$

subject to

$$R_t^s = \sum_{i=1}^{N} x_i \left(1 + r_{i,t}^s\right) \qquad \text{for all } t = 1..T \text{ and } s = 1..M \qquad (2.10)$$

$$\sum_{i=1}^{N} x_i = 1 \qquad (2.11)$$

$$x_i \geq 0 \qquad \text{for all } i = 1..N \qquad (2.12)$$

This model is different from the static model in two aspects. First, the return part (the first term) of the objective function (2.9) is defined as the expected annualized geometric return of the portfolio. The second difference can be observed in equation (2.10), where the total portfolio return per period is defined for each scenario.

3. Results

We generate 1000 equally likely return scenarios for each asset class for a period of ten years or forty quarters (Mulvey [1996 and 2000]). Using these return scenarios, we come up with the return-risk characteristics of each asset class in the opportunity set (shown in Table 3.1).

Asset Class	**Annualized Arithmetic Return (%)**	**Annualized Geometric Return (%)**	**Annualized Risk (Standard Deviation of Return) (%)**
Cash	4.92	5.01	0.36
European Equity	11.41	8.92	24.17
Far East Equity	11.67	9.11	24.74
Government Bonds	6.98	6.71	9.49
Lookback Straddles	18.29	5.06	59.49
Private Equity	11.71	9.41	23.78
US Equity	11.17	8.96	23.27
US High Quality Bonds	7.00	7.03	5.69

Table 3.1: Attributes of Asset Classes

For the static portfolio problem (Model 1), we find that the presence of lookback straddles in the opportunity set leaves the results unaltered. In other words, the lookback straddles do not improve the portfolio in a static portfolio optimization framework. Upon further examination, this is an intuitive conclusion, because for a *buy-and-hold* type outlook, an investor would not want to carry an asset class that has the properties of the lookback straddles (i.e. low geometric return and high volatility/risk).

The green line in Figure 3.1 depicts the efficient frontier for the single-period problem, with the corresponding mixes given in Table 3.2 (all numbers in percent).

Point	Cash	European equity	Far East Equity	Government Bonds	Lookback Straddles	Private equity	US Equity	US HQ Bonds	Return	Risk
1	99.91	0.00	0.00	0.00	0.00	0.09	0.00	0.00	5.02	0.36
2	77.31	0.00	0.00	0.00	0.00	1.40	0.00	21.29	5.50	1.48
3	54.18	0.00	0.00	0.00	0.00	2.76	0.00	43.06	6.00	2.80
4	31.16	0.00	0.00	0.00	0.00	4.20	0.00	64.64	6.50	4.06
5	8.24	0.00	0.00	0.00	0.00	5.73	0.00	86.03	7.00	5.26
6	0.00	0.53	2.21	0.00	0.00	15.36	2.48	79.42	7.50	7.24
7	0.00	4.34	5.54	0.00	0.00	29.17	4.02	56.93	8.00	10.95
8	0.00	8.12	8.99	0.00	0.00	42.72	5.78	34.39	8.50	14.96
9	0.00	11.42	12.28	0.00	0.00	55.36	9.30	11.64	9.00	19.02
10	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	9.41	23.78

Table 3.2. Efficient points for the static model

For the multi-period portfolio case (Model 2), we conduct the same strategy for generating an efficient frontier. Using a non-convex solver and varying values for α, we first restrict the weight for lookback straddles to be 0. The red line in Figure 3.1 depicts the resulting efficient frontier for this multi-period model, with the corresponding mixes given in Table 3.3 (all numbers in percent). Observe that the portfolios at the top of this frontier consist of multiple assets, as compared with the results of the static model. This result is common for multi-period investment models. See Mulvey (2000), and Fernholz and Shay (1982). Examination of the efficient portfolios shows the tendency to allocate more capital to private equity over public equity. This is due to its more attractive risk-return qualities, which do not take into account things such as liquidity and transaction costs (which we have ignored). In practical situations, these constraints would need to be included in the optimization model.

Point	Cash	European equity	Far East Equity	Government Bonds	Lookback Straddles	Private equity	US Equity	US HQ Bonds	Return	Risk
1	99.83	0.00	0.00	0.00	0.00	0.17	0.00	0.00	5.02	0.35
2	83.67	0.09	0.17	0.00	0.00	2.69	0.00	13.38	5.50	1.12
3	66.33	0.20	0.53	0.00	0.00	5.06	0.00	27.88	6.00	2.16
4	48.64	0.31	0.90	0.00	0.00	7.48	0.00	42.67	6.50	3.23
5	30.58	0.41	1.28	0.00	0.00	9.95	0.00	57.78	7.00	4.34
6	12.14	0.52	1.66	0.00	0.00	12.48	0.00	73.20	7.50	5.47
7	0.00	2.07	2.92	0.00	0.00	15.91	0.00	79.10	8.00	6.68
8	0.00	6.63	6.06	0.00	0.00	21.33	0.00	65.98	8.50	8.51
9	0.00	11.72	9.87	0.00	0.00	27.88	0.00	50.52	9.00	11.13

10	0.00	18.70	14.65	0.00	0.00	36.08	0.00	30.57	9.50	14.82
11	0.00	27.15	23.19	0.00	0.00	49.67	0.00	0.00	9.95	20.79

Table 3.3: Efficient points for the dynamic model without lookback straddles

Next, we remove the restriction on the lookback straddles, and thus include them in the opportunity set. The efficient frontier for this model is the blue line in Figure 3.1, along with the optimal mixes in Table 3.4 (all numbers in percent). We see that the lookback straddles appear in the portfolio at every point except for the minimum risk point. Indeed, we would not expect it to be visible in a minimum risk portfolio due to its high standard deviation.

Point	Cash	European equity	Far East Equity	Government Bonds	Lookback Straddles	Private equity	US Equity	US HQ Bonds	Return	Risk
1	99.83	0.00	0.00	0.00	0.00	0.17	0.00	0.00	5.02	0.35
2	83.85	0.00	0.12	0.00	0.10	2.66	0.00	13.28	5.50	1.12
3	66.81	0.00	0.39	0.00	0.26	4.98	0.00	27.56	6.00	2.15
4	49.41	0.00	0.67	0.00	0.41	7.35	0.00	42.16	6.50	3.23
5	31.65	0.00	0.94	0.00	0.58	9.77	0.00	57.07	7.00	4.33
6	13.55	0.00	1.21	0.00	0.74	12.24	0.00	72.26	7.50	5.46
7	0.00	0.00	2.09	0.00	1.39	15.25	0.00	81.26	8.00	6.63
8	0.00	1.35	4.23	0.00	3.35	19.63	0.00	71.44	8.50	8.29
9	0.00	2.95	6.49	0.00	5.61	24.62	0.00	60.34	9.00	10.55
10	0.00	4.66	9.30	0.00	8.37	30.30	0.00	47.36	9.50	13.47
11	0.00	6.83	12.56	0.00	12.13	37.51	0.00	30.96	10.00	17.41
12	0.00	9.92	18.19	0.00	19.80	49.41	0.00	2.69	10.50	24.77
13	0.00	5.77	18.50	0.00	24.80	50.93	0.00	0.00	10.54	26.82

Table 3.4. Efficient points for dynamic model with lookback straddles (1 Year, Vol. = 14.0%)

To compare results, we plot all three efficient frontiers on the same chart, depicted in Figure 3.1. The lookback options improve the performance of the multi-period investor by taking advantage of the asset's high volatility. Again, the top end of the efficient frontier consists of multiple assets, within the context of re-balancing rules.

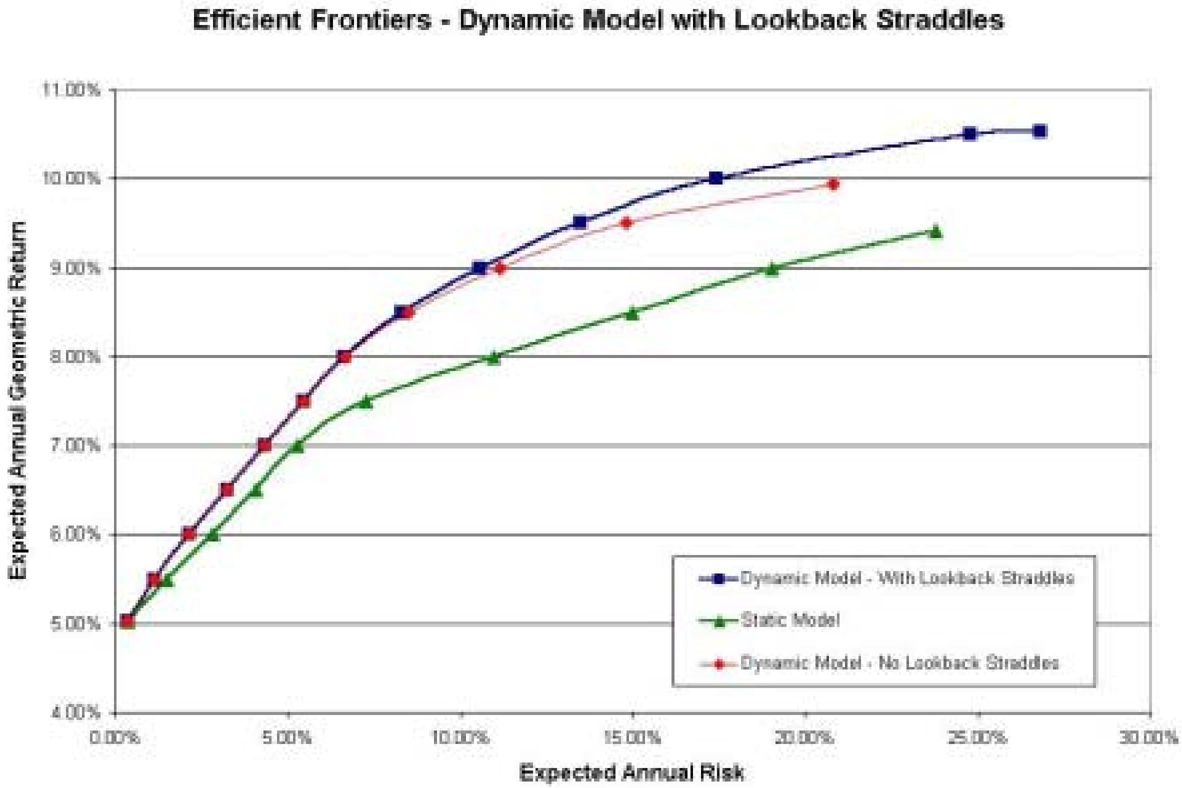

Figure 3.4. Efficient frontiers for all models

4. Conclusions

This article provides a starting point for studying how hedge funds can affect an investor's optimal investment strategy. By using a crude proxy, the lookback straddle, for a trend-following hedge fund, we find from simulation and numerical optimization that the presence of this high volatility, low return asset class can be used in a multi-period setting to improve the efficient frontier. In constructing the asset class, issues such as monitoring bias introduced by the discretization are dealt with by adjusting the volatility used in the Black-Scholes pricing formulas so that the asset class has the desired characteristics. The sensitivity to the choice of volatility is also an output of this analysis and is shown in Figure 2.4. This locates the pricing volatility of 14.7% as approximately the cost beyond which trend following will produce negative returns and will no longer improve the efficient frontier.

There are several directions for improvement: constructing a more accurate proxy for hedge funds, for example by employing other derivative securities calibrated from data; removing the restriction that feasible strategies are fixed-mix or buy-and-hold; incorporating transaction and market impact costs. In addition, CTAs invest across a range of asset classes, for example currencies and commodities, not just equities, so a future direction we have undertaken is to build a lookback asset class based on a wider range of underlying securities. The lookback approach can also be extended to model characteristics of hedge funds that might be focusing, for example, on the trend of a spread between two assets, rather than the trend of one underlying. This has been implemented in the context of convergence trades on credit spreads in Fung and Hsieh (2002) under the Black-Scholes pricing assumptions. For more complex fixed-income models, this would lead to interesting problems of pricing lookbacks on spreads.

This work is also a contribution to the literature on understanding how to trade optimally with derivatives in a realistic (incomplete) market. Here, simulations and stochastic programming methods provide some insight where the models are beyond the scope of existing analytical methods.

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Appendix A

Pricing Volatility	3 Months	6 Months	1 Year	2 Years	10 Years
11.0%	**Return = 0.4418 Risk = 1.49**	**Return = 0.4797 Risk = 1.02**	**Return = 0.321 Risk = 0.7127**	**Return = 0.2217 Risk = 0.5417**	**Return = 0.1252 Risk = 0.3191**
11.5%	**Return = 0.2104 Risk = 1.4262**	**Return = 0.3576 Risk = 0.9799**	**Return = 0.267 Risk = 0.6894**	**Return = 0.1979 Risk = 0.5285**	**Return = 0.1221 Risk = 0.3154**
12.0%	**Return = 0.0234 Risk = 1.3677**	**Return = 0.2499 Risk = 0.943**	**Return = 0.217 Risk = 0.6677**	**Return = 0.1753 Risk = 0.5161**	**Return = 0.1191 Risk = 0.3118**
12.5%	Return = -0.1288 Risk = 1.3137	**Return = 0.1544 Risk = 0.9091**	**Return = 0.1708 Risk = 0.6475**	**Return = 0.1538 Risk = 0.5044**	**Return = 0.1161 Risk = 0.3085**
13.0%	Return = -0.2539 Risk = 1.2638	**Return = 0.0693 Risk = 0.8777**	**Return = 0.1279 Risk = 0.6288**	**Return = 0.1334 Risk = 0.4934**	**Return = 0.1131 Risk = 0.3052**
13.5%	Return = -0.3573 Risk = 1.2175	Return = -0.0068 Risk = 0.8487	**Return = 0.0879 Risk = 0.6112**	**Return = 0.114 Risk = 0.483**	**Return = 0.1102 Risk = 0.3021**
14.0%	Return = -0.4435 Risk = 1.1745	Return = -0.075 Risk = 0.8219	**Return = 0.0506 Risk = 0.5949**	**Return = 0.0954 Risk = 0.4732**	**Return = 0.1073 Risk = 0.2992**
14.5%	Return = -0.5157 Risk = 1.1343	Return = -0.1366 Risk = 0.7969	**Return = 0.0157 Risk = 0.5796**	**Return = 0.0777 Risk = 0.4639**	**Return = 0.1044 Risk = 0.2963**
15.0%	Return = -0.5766 Risk = 1.0969	Return = -0.1922 Risk = 0.7738	Return = -0.017 Risk = 0.5652	**Return = 0.0608 Risk = 0.455**	**Return = 0.1016 Risk = 0.2936**
15.5%	Return = -0.6283 Risk = 1.0618	Return = -0.2426 Risk = 0.7522	Return = -0.0477 Risk = 0.5518	**Return = 0.0446 Risk = 0.4466**	**Return = 0.0989 Risk = 0.2911**
16.0%	Return = -0.6723 Risk = 1.0289	Return = -0.2885 Risk = 0.7321	Return = -0.0766 Risk = 0.5391	Return = 0.0291 Risk = 0.4386	**Return = 0.0962 Risk = 0.2886**
16.5%	Return = -0.71 Risk = 0.9979	Return = -0.3304 Risk = 0.7134	Return = -0.1038 Risk = 0.5273	Return = 0.0143 Risk = 0.431	**Return = 0.0935 Risk = 0.2862**
17.0%	Return = -0.7424 Risk = 0.9687	Return = -0.3687 Risk = 0.6959	Return = -0.1295 Risk = 0.5161	Return = 0.0000 Risk = 0.4238	**Return = 0.0909 Risk = 0.284**
17.5%	Return = -0.7704 Risk = 0.9412	Return = -0.4038 Risk = 0.6795	Return = -0.1537 Risk = 0.5056	Return = -0.0137 Risk = 0.4169	Return = 0.0883 Risk = 0.2818
18.0%	Return = -0.7948 Risk = 0.9152	Return = -0.4361 Risk = 0.6643	Return = -0.1767 Risk = 0.4958	Return = -0.0269 Risk = 0.4103	Return = 0.0858 Risk = 0.2798
18.5%	Return = -0.816 Risk = 0.8906	Return = -0.4659 Risk = 0.65	Return = -0.1985 Risk = 0.4865	Return = -0.0396 Risk = 0.4041	Return = 0.0833 Risk = 0.2778

Table A.1: Properties of different lookback straddle asset classes. Those highlighted in green are those that most resemble those of the PTFS used by Fung and Hsieh and will affect the efficient frontier by way of volatility pumping. Those highlighted in red would also affect the efficient frontier but purely because they have high returns.

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